CIS 2166, Introduction to Finite State Machines

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Notes to Chapter 13.2 of Rosen's textbook. Introduces Finite State Machines with Output. Gives a definition and an example that is simpler than the one found in the textbook.

A Formal Definition of a Finite State Machine with Output

A Finite State Machine with Output, M, is the following 6-tuple:

$M = (S,$ `// finite set of states`

$\quad I,$ `// finite input alphabet`

$\quad O,$ `// finite output alphabet`

$\quad f,$ `//transition function (assigns to each (state, input) pair a new state`

$\quad g,$ `//output function (assigns to each (state, input) pair an output`

$\quad S_0)$ `// designated START state`

Example

Consider a coke vending machine, similar to the one described in the textbook, only simpler.

This machine is perfectly suited to be modeled by the 'Finite-State-Machine-with-Output' model just defined and has the following features:

- Accetps only quarters
- A drink costs 75 cents
- Has no `'Return My Change'` button
- Only vends Coke, Orange Juice, or Beer

How does the machine work? Can the above formal definition of a finite state machine be understood with this machine?

- *States* represent how close you are to getting your drink - a state represents how many quarters are sitting in the machine at any given moment.

  There could be 3, 2, 1, or 0 quarters in this machine at any given time, and only 3 quarters will buy you a drink.

  Call the states $(S_0, S_{25}, S_{50}, S_{75})$, with S_0 meaning 'no quarters currently in the machine', S_{25} - 'exactly one quarter in the machine',

Figure 1: A Finite State Machine with Output.

S_{50} - 'two quarters', and S_{75} - 'three quarters currently in the machine'.

- *Inputs* are the actions that you undertake. These are 'deposit a quarter', for example; or 'press a button to vend a drink'.

  Use $I = (¢25, C, O, B)$ to denote the inputs for this machine. Here B stands for 'press the beer button', O stands for 'press the orange juice button', C for 'press the coke button' and ¢25 stands for 'deposit a quarter into the machine'.

- *Outputs* are $(Coke, OJ, Beer, \emptyset)$, where $\emptyset$ means that you get nothing out of the machine and $Coke, OJ$, and $Beer$ stand for the drinks you can get out of the machine.

- *Transition and Output Functions*.

  When in a given state (think: three quarters in the machine right now), and on a given input (think: about to press the beer button), the machine transits to another state (think: no more quarters in the machine soon) and gives you an output (think: your much awaited can of beer is about to be vended). Thus for each state-input pair we transit to another state and receive some output (which maybe empty, denoted by $\emptyset$).

  It is customary to specify these transition functions in a table, or by a state diagram. We'll show both.

	Input				Output			
State	¢25	C	O	B	¢25	C	O	B
S_0	S_{25}	S_0	S_0	S_0	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
S_{25}	S_{50}	S_{25}	S_{25}	S_{25}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
S_{50}	S_{75}	S_{50}	S_{50}	S_{50}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
S_{75}	S_{75}	S_0	S_0	S_0	¢25	Coke	OJ	Beer

  For example if you are in state S_{75} (you've given up three quarters to the machine), and you input another quarter, the output will be a quarter (we assume the machine doesn't swallow your extra quarters). On the other hand, in the same state, if you input C (i.e. you press the 'Coke' button) then the output will be C (i.e. the machine will vend you a coke). You find this in the table by following the state row for S_{75} to the right, toward the output part of the table, and looking in the entry for the column labeled ¢25 or C in that part of the table.

- Finally, specify the `START` state. We'll assume S_0 to be the start state: as you approach the machine and begin to contemplate which type of refreshment might serve best to lift your spirits, no one before you has been generous enough to leave any money in the machine for you.

- Alternatively, the machine can be represented pictorially. Each edge in the following diagram is labeled by an input-output pair. For example, the edge from S_0 to S_1 is labeled with (¢25, $\emptyset$), meaning that when in state S_0, and input of ¢25 gets you nothing. But the edge labeled $(C, Coke)$ means that pressing the C button easily quenches your thirst with the sugary drink when the machine is in state S_{75}.

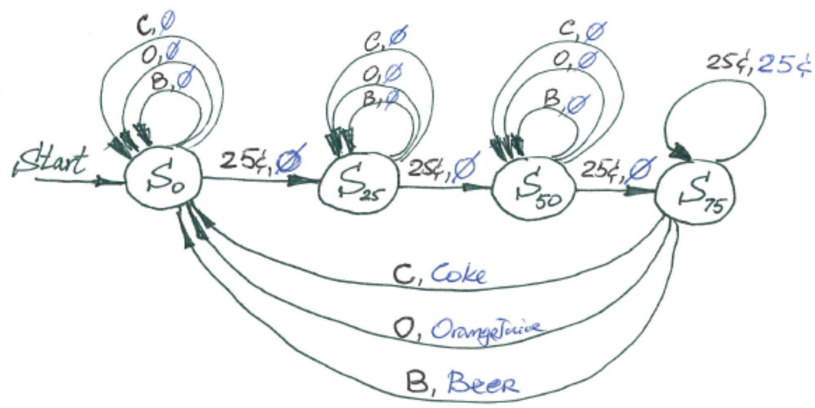

Figure 2: *A State Diagram Representing the Vending Machine.*