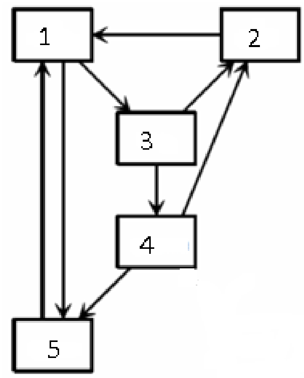
1. Determine whether the wheel graph W_3 and the complete graph K_4 are isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

2. For the web graph shown below, write the link matrix A that expresses the system of PageRank linear equations in the form $\mathbf{Ax} = \mathbf{x}$, where $\mathbf{x} = [x_1\ x_2\ x_3\ x_4\ x_5]^T$.
Is the matrix $\mathbf{M} = (1 - m)\mathbf{A} + m\mathbf{S}$ for m=0.25 column-stochastic? Justify your answer.



3. Given the wheel graph W_3, where the center node *a* links to three other nodes *b, c,* and *d*. Which of the following is true about the PageRank importance score of node *a* compared to *b, c*, and *d*? Write a justification for your answer.

A. *a* has the highest importance score
B. *a* has the lowest importance score
C. All nodes have equal importance scores
D. Each of *b, c*, and *d* has a higher importance score than *a*.

4. Use induction to prove that every amount of postage of n cents greater than 5 cents can be formed from 3-cent and 4-cent stamps.

5. Use the method of Gaussian elimination to find **x** for the system of linear equations **Ax**=**b**, where **A** and **b** are given below. Show your work.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 2 & 6 & 11 \end{bmatrix}, b = \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix}$$

6. Use the method of Gaussian elimination to find the determinant of matrix **B** given below. Show your work.

$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 3 \\ 2 & -2 & 0 \end{bmatrix}$$

7. Find the eigenvalues and the eigenvectors of these two matrices. Show your work.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad A + I = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$$

8. Find the eigenvalues and the eigenvectors of matrix A. Show your work.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 3 & 3 \end{bmatrix}.$$

9. Find the matrix **A** that performs those transformations, in order, on the Cartesian plane. To which point is the point (-4, 1) mapped by this transformation?
(a) horizontal stretch by a factor of 2
(b) reflection across the y-axis.

10. Find the standard matrix **A** for the given linear transformation T.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 - 3x_3 \\ 0 \\ x_1 + 4x_3 \\ 5x_2 + x_3 \end{bmatrix}$$

11. Provide a pseudo-code of an algorithm for finding the second largest number in a sequence of n distinct integers ($n > 1$) distinct integers. What is its worst-case time complexity in terms of the number of comparisons? Justify your answer.

12. Consider a single-layer perceptron with input vector **x** and weight vector **w**. The activation function is ReLU $g(z) = \max\{0, z\}$. Compute the output value for **x** and weight vector **w** given below. Show your work.

x= [1, 0, 1], **w** = [2, -1.6, -2.5].

13. Consider a 2-layer neural network structured as follows:

- **Input layer**: 2 input neurons: $x_1=1$ and $x_2=2$
- **Hidden layer**: 2 neurons and ReLU activation
- **Output layer**: 1 neuron, no activation (i.e., linear output)

What is the final output of the network for the weights and biases given below? Show your work.

- Hidden layer:

$$W^{[1]} = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}, \quad b^{[1]} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- Output layer:

$$W^{[2]} = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad b^{[2]} = 0$$

14. Use strong induction to show that every positive integer can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, and so on. (Hint: For the inductive step, separately consider the case where $k + 1$ is even and where it is odd. When it is even, note that $(k + 1)/2$ is an integer.)