CIS 2166 Fall 2013 Homework 1 on Matrix Algebra

1.

Solve the following system of equations by elimination and back substitution:

$$\begin{cases} x & +2y & +3z & = & 0 \\ -x & +y & -2z & = & -3 \\ 2x & +y & +z & = & 3 \end{cases}$$

Let $Ax = b$ denote the above system. Write the three elimination matrices E_{21}, E_{31} and E_{32} that put A into triangular form U with $E_{32}E_{31}E_{21}A = U$.

Compute $M = E_{32}E_{31}E_{21}$.

2.

Find a combination $x_1w_1 + x_2w_2 + x_3w_3$ that gives the zero vector:

$$w_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} ; \; w_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} ; \; w_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} .$$

Those vectors are (independent)(dependent). The three vectors lie in a ________. The matrix W with those columns is *not invertible.*

3.

Find a solution for x,y,z to the system of equations

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\,e + \pi + 2\sqrt{2} \\ 6\,e + 4\,\pi + 5\sqrt{2} \\ 10\,e + 7\,\pi + 8\sqrt{2} \end{pmatrix}$$

4.

Find the linear combination $2\boldsymbol{s}_1 + 3\boldsymbol{s}_2 + 4\boldsymbol{s}_3 = \boldsymbol{b}$. Then write $\boldsymbol{b}$ as a matrix-vector multiplication $S\boldsymbol{x}$. Compute the dot products (row of S) $\cdot\, \boldsymbol{x}$:

$$\boldsymbol{s}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \boldsymbol{s}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad \boldsymbol{s}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ go into the columns of S .

5.

Find a combination $x_1 \boldsymbol{w}_1 + x_2 \boldsymbol{w}_2 + x_3 \boldsymbol{w}_3$ that gives the zero vector:

$$\boldsymbol{w}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \boldsymbol{w}_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad \boldsymbol{w}_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}.$$

Those vectors are (independent) (dependent). The three vectors lie in a _____. The matrix W with those columns is *not invertible*.

6.

Which values of c give dependent columns (combination equals zero)?

$$\begin{bmatrix} 1 & 3 & 5 \\ 1 & 2 & 4 \\ 1 & 1 & c \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & c \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} c & c & c \\ 2 & 1 & 5 \\ 3 & 3 & 6 \end{bmatrix}$$

7.

Find the inverses (directly or from the 2 by 2 formula) of A, B, C:

$$A = \begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 0 \\ 4 & 2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}.$$

8.

Solve for the first column (x, y) and second column (t, z) of A^{-1}:

$$\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$