Solutions by Archana Gupta

<u>Question 1 (</u>(Chapter 3: 10)
An 8-bit byte with binary value 10101111 is to be encoded using an even-parity Hamming code. What is the binary value after encoding?

<u>Answer</u>
Check bits are inserted at positions that are powers of 2 i.e. 1,2,4,8,16,32,e.t.c. Data bits are at positions 3,5,6,7,9,10,11,12 e.t.c. So after inserting check bits our data should look like this:

	?	**?**	1	**?**	0	1	0	**?**	1	1	1	1
positions	**1**	**2**	3	**4**	5	6	7	**8**	9	10	11	12

3 = 1+2
5 = 1+4
6 = 2+4
7 = 1+2+4
9 = 1+8
10 = 2+8
11 = 1+2+8
12 = 4+8

Hence for the check bit 1 we look at bits 3,5,7,9,11 and get value 1.
For check bit 2 we look at bits 3,6,7,10,11 and get value 0.
For check bit at position 4 we look at bits 5,6,7,12 and get value 0.
For check bit at position 8 we look at bits 9,10,11,12 and get value 0.

Hence the binary value after encoding is **1 0** 1 **0** 0 1 0 **0** 1 1 1 1.

<u>Question 2 (</u>Chapter 3: 15)
A bit stream 10011101 is transmitted using the standard CRC method. The generator polynomial is $x^3 + 1$. Show the actual bit string transmitted. Suppose the third bit from the left is inverted during transmission. Show that this error is detected at the receivers end.

<u>Answer</u>
Our generator $G(x) = x^3 + 1$ encoded as 1001. Because the generator polynomial is of the degree three we append three zeros to the lower end of the frame to be transmitted. Hence after appending the 3 zeros the bit stream is **10011101000**. On dividing the message by generator after appending three zeros to the frame we get a remainder of 100. We do modulo 2 subtraction thereafter of the remainder from the bit stream with the three zeros appended. **The actual frame transmitted is 10011101100**. See below.

```
               1 0 0 0 1 1 0 0
      -------------------------
1001 | 1 0 0 1 1 1 0 1 0 0 0
     | 1 0 0 1
       -------
         0 0 0 1
         0 0 0 0
         -------
           0 0 1 1
           0 0 0 0
           -------
             0 1 1 0
             0 0 0 0
             -------
               1 1 0 1
               1 0 0 1
               -------
                 1 0 0 0
                 1 0 0 1
                 -------
                   0 0 1 0
                   0 0 0 0
                   -------
                     0 1 0 0
                     0 0 0 0
                     -------
                       1 0 0 (remainder)
```

Actual frame transmitted : 10011101000 – 100 = 10011101100 (modulo 2 subtraction)

Now suppose the third bit from the left is garbled and the frame is received as 10111101100. Hence on dividing this by the polynomial generator we get a remainder of 100 which shows that an error has occurred. Had the received frame been error free we would have got a remainder of zero. See below.

```
                1 0 1 0 1 0 0 0
     ----------------------------
1001| 1 0 1 1 1 1 0 1 0 0 0
    | 1 0 0 1
      -------
        0 1 0 1
        0 0 0 0
        -------
          1 0 1 1
          1 0 0 1
          -------
            0 1 0 0
            0 0 0 0
            -------
              1 0 0 1
              1 0 0 1
              -------
                0 0 0 1
                0 0 0 0
                -------
                  0 0 1 0
                  0 0 0 0
                  -------
                    0 1 0 0
                    0 0 0 0
                    -------
                      1 0 0 (remainder indicating error)
```