

# Stochastic Simulation

- **Idea:** probabilities  $\leftrightarrow$  samples
- Get probabilities from samples:

| $X$          | <i>count</i> |
|--------------|--------------|
| $x_1$        | $n_1$        |
| $\vdots$     | $\vdots$     |
| $x_k$        | $n_k$        |
| <i>total</i> | $m$          |

 $\leftrightarrow$ 

| $X$      | <i>probability</i> |
|----------|--------------------|
| $x_1$    | $n_1/m$            |
| $\vdots$ | $\vdots$           |
| $x_k$    | $n_k/m$            |

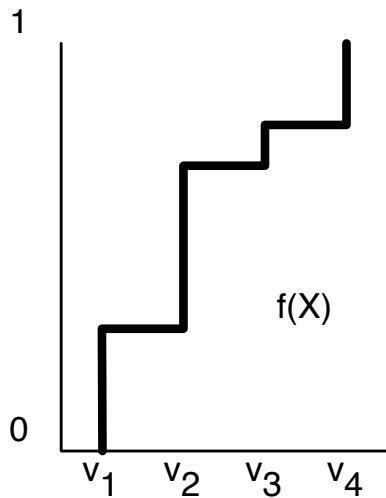
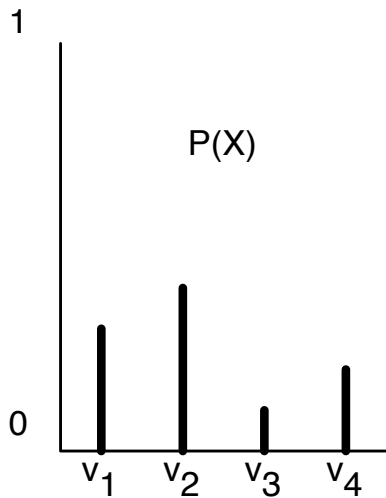
- If we could sample from a variable's (posterior) probability, we could estimate its (posterior) probability.

# Generating samples from a distribution

For a variable  $X$  with a discrete domain or a (one-dimensional) real domain:

- Totally order the values of the domain of  $X$ .
- Generate the cumulative probability distribution:  
 $f(x) = P(X \leq x)$ .
- Select a value  $y$  uniformly in the range  $[0, 1]$ .
- Select the  $x$  such that  $f(x) = y$ .

# Cumulative Distribution



# Forward sampling in a belief network

- Sample the variables one at a time; sample parents of  $X$  before you sample  $X$ .
- Given values for the parents of  $X$ , sample from the probability of  $X$  given its parents.

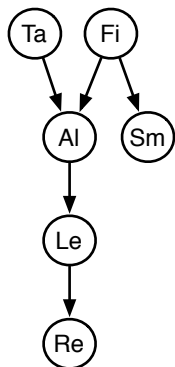
# Rejection Sampling

- To estimate a posterior probability given evidence  $Y_1 = v_1 \wedge \dots \wedge Y_j = v_j$ :
- Reject any sample that assigns  $Y_i$  to a value other than  $v_i$ .
- The non-rejected samples are distributed according to the posterior probability:

$$P(\alpha) \approx \frac{\sum_{sample \models \alpha} 1}{\sum_{sample} 1}$$

where we consider only samples consistent with observations.

# Rejection Sampling Example: $P(ta|sm, re)$



|            | Ta    | Fi    | Al    | Sm    | Le    | Re    |   |
|------------|-------|-------|-------|-------|-------|-------|---|
| $s_1$      | true  | false | true  | false | —     | —     | ✗ |
| $s_2$      | false | true  | false | true  | false | false | ✗ |
| $s_3$      | false | true  | true  | true  | true  | true  | ✓ |
| $s_4$      | true  | true  | true  | true  | true  | true  | ✓ |
| ...        |       |       |       |       |       |       |   |
| $s_{1000}$ | false | false | false | false | —     | —     | ✗ |

$$P(sm) = 0.02$$

$$P(re|sm) = 0.32$$

How many samples are rejected?

How many samples are used?

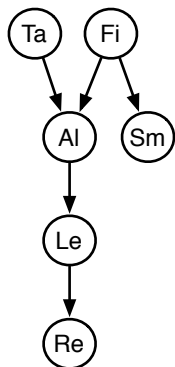
# Importance Sampling

- Samples have weights: a real number associated with each sample that takes the evidence into account.
- Probability of a proposition is weighted average of samples:

$$P(\alpha | observations) \approx \frac{\sum_{sample \models \alpha} weight(sample)}{\sum_{sample} weight(sample)}$$

- If we can compute  $P(evidence | sample)$  we can weight the (partial) sample by this value.
- Mix exact inference with sampling: don't sample all of the variables, but weight each sample appropriately.
- Sample according to a proposal distribution, as long as the samples are weighted appropriately.

# Importance Sampling Example: $P(ta|sm, re)$



|            | Ta    | Fi    | Al    | Le    | Weight             |
|------------|-------|-------|-------|-------|--------------------|
| $s_1$      | true  | false | true  | false | $0.01 \times 0.01$ |
| $s_2$      | false | true  | false | false | $0.9 \times 0.01$  |
| $s_3$      | false | true  | true  | true  | $0.9 \times 0.75$  |
| $s_4$      | true  | true  | true  | true  | $0.9 \times 0.75$  |
| ...        |       |       |       |       |                    |
| $s_{1000}$ | false | false | true  | true  | $0.01 \times 0.75$ |

$$P(sm|fi) = 0.9$$

$$P(sm|\neg fi) = 0.01$$

$$P(re|le) = 0.75$$

$$P(re|\neg le) = 0.01$$



# Particle Filtering



- Suppose the evidence is  $e_1 \wedge e_2$   
 $P(e_1 \wedge e_2 | sample) = P(e_1 | sample)P(e_2 | e_1 \wedge sample)$
- After computing  $P(e_1 | sample)$ , we may know the sample will have an extremely small probability.
- Idea: we use lots of samples: “particles”. A particle is a sample on some of the variables.
- Based on  $P(e_1 | sample)$ , we resample the set of particles. We select from the particles according to their weight.
- Some particles may be duplicated, some may be removed.