Quantized Conflict Graphs for Wireless Network Optimization

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***Abstract*—Conflict graph has been widely used for wireless network optimization in dealing with the issues of channel assignment, spectrum allocation, links scheduling and etc. Despite its simplicity, the traditional conflict graph suffers from two drawbacks. On one hand, it is a rough representation of the interference condition, which is inaccurate and will cause sub-optimal results for wireless network optimization. On the other hand, it only defines the interference between two entities, which neglects the accumulative effect of small amount interference. In this paper, we propose the model of quantized conflict graph (QCG) to tackle the above issues. The properties, usage and construction methods of QCG are explored. We show that in its matrix form, a QCG owns the properties of low-rank and high-similarity. These properties give birth to three complementary QCG estimation strategies, namely low-rank approximation approach, similarity based approach, and comprehensive approach, to construct the QCG efficiently and accurately from partial interference measurement results. We further explore the potential of QCG for wireless network optimization by applying QCG in minimizing the total network interference. Extensive experiments using real collected wireless network are conducted to evaluate the system performance, which confirm the efficiency of the proposed algorithms.**

***Index Terms*—Wireless network optimization, Interference model, Conflict graph, Matrix completion**

I. Introduction

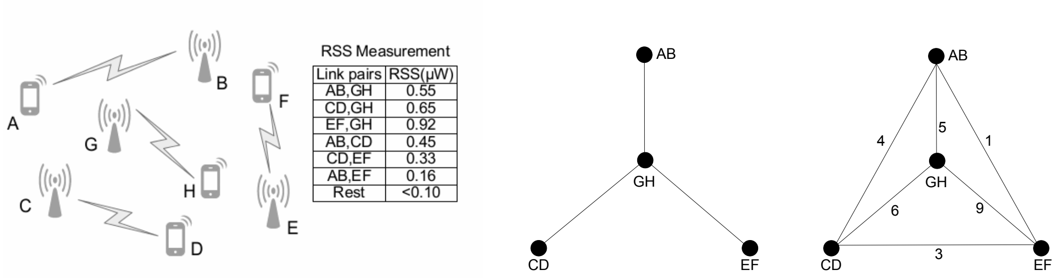
The performance of wireless networks could be improved by optimizing the allocation of existing wireless spectrum resources via link scheduling, channel allocation, etc. [1], [2]. Such optimizations usually allow multiple nodes to transmit at the same time over the same channel, while dealing with the interference of simultaneous transmissions appropriately. Two main interference models have been proposed in the literature [3]: the protocol model and the physical interference model, also known as the Signal to Interference and Noise Ratio (SINR) model.

The SINR model is considered to be a realistic interference model. It accurately models the accumulative interference effect, which refers to the interference aggregated from multiple sources. However, the SINR model suffers from some problems such as high cost caused by exhaustive measurement calibration. More importantly, its non-convex nature incurs great complexity when it is applied in wireless network optimization.

The protocol interference model or so-called conflict graph is a simple graphical representation of the interference condition between any two wireless communication links (or wireless nodes). In such a model, if the interference between entities exceeds a threshold, they are considered "conflict" and there will be an edge linking the two entities in the conflict graph. Conflict graph provides a simplified description of the interference status, which greatly eases the design of channel assignment/spectrum allocation algorithms, and consequently gives birth to a series of highly efficient wireless network optimization algorithms [1], [4]–[6].

However, there are two major drawbacks of the conflict graph. First, the conflict graph is a coarse-grained and rough estimation of the interference condition. It can only indicate whether the interference between a pair of entities exceeds a threshold, but it cannot describe the degree of the interference. Due to the fact that interference is dynamic and fluctuating, it can hardly be depicted by a simple threshold. Especially when interference fluctuates around the threshold, the corresponding conflict graph will be dynamic in the structure. As suggested by the studies of [7], [8], such binary representation oversimplifies the communication condition, which makes the conflict graph highly inaccurate and will consequently compromise the wireless network optimization results [9]. Second, the conflict graph only defines the interference between two entities, but neglects the small amount interference from a third node. However, such small amount interference accumulated from multiple concurrent transmitters in the same channel could also cause severe communication failures [7], [10].

In this paper, we propose the model of quantized conflict graph (QCG) to address the above issues. In this model, the interference between entity pairs is quantized to an M-level discrete measurement. The M-level quantization is a finer-grain estimation of the interference condition, which can capture the degree of interference to make a more accurate presentation. With the proposed M-level quantization, traditional conflict graph becomes a special case of quantized conflict graph with $M = 2$. Furthermore, quantized conflict graph captures the minor interference between nodes, which helps to identify accumulative interference in wireless network environment. In order to get an accurate QCG, the measurements of RSS (Receiving Signal Strength) between each node are required. However, the measurement method is too costly to be applied. To avoid exhaustive measurement, we propose estimation methods to construct a QCG from partial interference measurement. We treat the QCG as a matrix with some entries known while the others are not. Thus, QCG construction



(a) A scenario of 4 links. (b) Conflict graph. (c) Quantized conflict graph.

Fig. 1: An example of quantized conflict graph representation.

is modeled as a matrix completion problem. Based on the low-rank property of the quantized interference matrix and the high correlation between the interference measurements, several strategies are proposed to estimate the QCG matrix in an efficient way.

We also present the usage of QCG to improve the wireless network optimization results. Specifically, we study the QCG-based channel assignment problem in order to minimize the overall communication interference in a wireless network. We show that this problem equals to the classical Max K-cut problem and can be solved with efficient approximate algorithms.

Extensive experiments using real collected wireless network are conducted to evaluate the performance of the proposed algorithms. The results show that the proposed QCG estimation strategies achieve low estimation error for proper system settings, and with the application of QCG, the interference minimization in wireless network approaches the upper bound of performance gain.

The contributions of this paper are summarized as follows:

- **A quantized conflict graph model.** We propose the quantized measurement of interference, the formulation of QCG, and the definition of quantized RSS matrix, which provides finer-grain representation of interference in wireless networks.
- **Properties of QCG.** By analysing two datasets collected from indoor and outdoor environment, we explore several properties of QCG such as low-rank matrix and high-similarity among the interference measurements.
- **Strategies for efficient QCG estimation.** We introduce several strategies to estimate the QCG matrix from partial measurement, namely low-rank approximation approach, similarity based approach, and comprehensive approach, which are shown to achieve high estimation accuracy and efficiency.
- **QCG-based wireless network optimization.** We explore the potential of QCG for wireless network optimization by applying QCG for channel assignment to minimize network interference. We show that QCG-based interference minimization problem is equal to the Max K-cut problem and can be solved efficiently.

The rest of papers are organized as follows: In Section II we present the motivation and the system models. We then depict the experimental findings from two real datasets in Section III. In Section IV, complementary efficient QCG construction solutions are presented. In Section V, we use the network interference minimization problem to illustrate the advantage and usage of QCG. The performance results are stated in Section VI. Finally, we state the related work in Section VII and conclude this paper in Section VIII.

II. Motivation and Definition

In this section, we present the motivation and definition of QCG.

A. Motivation

Quantized conflict graph (QCG) is a graph with discrete indicators on the edges to depict the level of interference for simultaneous wireless transmissions. An example of QCG is illustrated in Fig. 1. Fig. 1(a) shows the scenario of four wireless links and their measured interference by RSS (in μW). If applying a threshold of 0.5 μW, the traditional conflict graph is obtained as Fig. 1(b), which shows that link GH conflicts with AB, CD and EF for parallel transmission. If we use the scales from 0 to 9 to denote the interference level, the quantized conflict graph is shown in Fig. 1(c). Traditional conflict graph is a special case of quantized conflict graph with $0-1$ level of conflict representation.

The motivation of using a quantized conflict graph lies in twofold. First, traditional conflict graph uses a binary conflict relationship to characterize the interference, which ignores the minor interference accumulated from co-channel communication pairs. Due to the fact that the conflict graph oversimplifies the interference relationship by omitting the level of interference and the accumulative interference, it may cause sub-optimal channel assignment in wireless networks, which can be leveraged by using the quantized conflict graph. Second, the quantized conflict graph can improve the accuracy of interference estimation with low system overhead of link measurement. Specifically, interferences in QCG can be estimated efficiently by the way of "measure a few and predict a lot", which can be applied for wireless network optimization such as interference minimization to achieve near-optimal results.

B. M-level quantized RSS measurement

Receiving signal strength (RSS) is the most straightforward indicator to express the level of interference between nodes/links. In 802.11x networks, the RSS measurement can be directly obtained from the received packets' field of RSSI (Receiving Signal Strength Indicator). We consider a M-level quantized RSS measurement as following.

Assume C_{max} is the maximum measured power (in μW). We divide the range $[0, C_{max}]$ into M equal intervals numbered 0 through $M-1$, which indicates M different interference levels where 0 represents no interference and a larger number corresponds to a higher interference degree in wireless communication.

Specifically, we can apply a step function f to map the measured RSS value to a quantized value such as

$$f(x) = \lfloor \frac{x \times M}{C_{max}} \rfloor. \quad (1)$$

It is easy to verify that the quantized RSS measurement has the following properties.

Property 1: $\forall x_1, x_2 \in [0, C_{max}]$, if $x_1 < x_2$ then $f(x_1) \leq f(x_2)$;

Property 2: $\forall x_1, x_2 \in [0, C_{max}]$, if $f(x_1) < f(x_2)$ then $x_1 < x_2$.

Such properties suggest that the quantized value can be taken as a coarse estimation of the real RSS measurement and it is linear additive, which is useful for wireless network optimization such as interference minimization. In the real world, RSS measurement is dynamic and varies over time. Thus the quantized RSS should represent the mean of the RSS measurement in long period.

C. Quantized conflict graph

In a typical wireless network, a set of wireless devices are deployed and communicate with each other simultaneously. A quantized conflict graph is a graphical representation of the mutual interference degree between pairs of nodes/links in the wireless network. The general definition of QCG is given below.

Definition 1: (***Quantized Conflict Graph***) A QCG is defined as a graph $G_q = (V_q, E_q, W_q)$, where $V_q = \{1, 2, \cdots, N\}$ is a set of nodes/links in the wireless network, and $E_q = \{(i, j) | i \in V_q, j \in V_q\}$ represents a conflict between two neighbouring vertex. The set $W_q = \{w_{i,j} | (i, j) \in E_q, q_{i,j} \in \{1, 2, \cdots, M\}\}$, where $w_{i,j}$ denotes the non-zero quantized RSS measurement regarding the vertex pair i and j.

D. Quantized RSS matrix

The quantized RSS matrix is the matrix representation of a quantized conflict graph, which is defined below.

Definition 2: (***Quantized RSS matrix***) Given a conflict graph $G_q = (V_q, E_q, W_q)$ with size N, the quantized-RSS matrix $Q = [q_{ij}]_{N \times N}$ is an $N \times N$ matrix with each elements q_{ij} being the quantized RSS measurement of vertex i and j.

According to the definition, $q_{ij} = w_{ij}$ if $(i, j) \in E_q$, and $q_{ij} = 0$ for the rest. Generally speaking, the quantized conflict graph is not a complete graph, thus Q is a sparse matrix with many elements equals to 0.

TABLE I: Notations

Notations	Explanation
G_q	quantized Conflict Graph
V_q	Vertices in QCG
E_q	Edges in QCG
W_q	quantized Weights in QCG
(u, v)	Edge between u and v
$\hat{X}$	Estimated RSS matrix
x_{ij}	Measured RSS between i and j
$\hat{x}_{ij}$	Estimated RSS between i and j
Ω	The measured set of node pairs
$\|\|X\|\|_*$	The nuclear norm of X
$\|\|X\|\|_F$	The Frobenius norm of X
$\|\|X\|\|_1$	The ℓ_1 norm of X
$\mathcal{C}$	The set of quantized values
γ, λ	The regularized constant in
$S(i, j)$	Similarity between rows/columns i and j
w_k	The weights to compute unmeasured RSS
$I(i, j)$	Assignment indicator of j channel to node i

Since quantized RSS matrix is simply the matrix representation of QCG, in the rest of this paper, we will use the terms of quantized RSS matrix and QCG alternatively without distinction.

The notations used in the paper are summarized in Table I.

III. Properties of QCG

Our basic idea to efficiently generate QCG, or in other words, to obtain quantized RSS matrix, is to follow a "measure a few and predict a lot" framework. Before presenting the methods in detail, we explore the properties of the quantized RSS matrix through real datasets analysis.

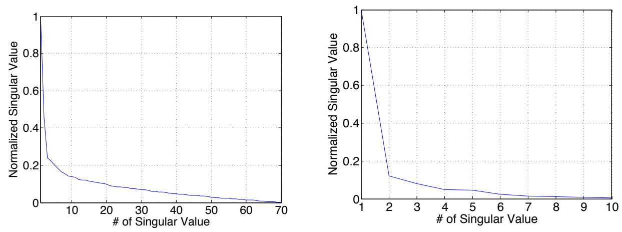
A. Datasets

We use two datasets collecting from indoor and outdoor wireless communication environment to analyse the properties of quantized RSS matrix.

SWIM (indoor) [9]. It consists of 10 wireless nodes running in 802.11a/b/g mode, which were deployed in the $3rd$ floor of a building. The data of the RSSI of the beacons were collected from each AP. Specifically, it activated one node at a time, while each AP was tuned to 11 different channels sequentially. Then, the collector walked to 25 different locations (including the locations of 10 AP), to collect the 50 different beacon messages from one AP in each channel. The RSSI, AP ID, and channel ID were recorded.

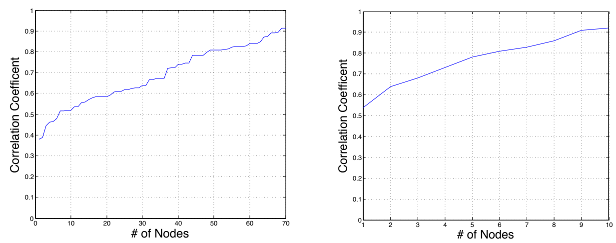
MetroFi (outdoor) [11]. This dataset consists of RSS values in a $7km^2$ area of an 802.11x municipal network in Portland, Oregon. It was collected by a research group in 2007. This dataset covers 30,991 distinct measured locations of 70 APs with known GPS locations and generates more than 200,000 samples.

The MetroFi dataset did not collect the interference RSS at the APs' location, thus to derive the interference relationship between nodes, we use the average of top-k RSS in each AP's region. Specifically, we set the region as the circle with radius of 50 meters, and set k as 5. For the SWIM dataset, the RSS values were collected in the position of APs, thus it is much easier to get a RSS matrix from the packet heads. The RSS values are in form of energy power with unit of Walt. The RSS measurements are quantized using the method introduced in Section II-B for further analysis.



(a) MetroFi dataset. (b) SWIM dataset.

Fig. 2: Singular values of the quantized RSS matrix.



(a) MetroFi dataset. (b) SWIM dataset.

Fig. 3: Correlation coefficient of rows of the quantized RSS matrix.

B. Properties

We use Matlab to analyse the properties of the quantized RSS matrix in the following.

1) Low rank property: Firstly, we analyse the rank property of the RSS matrices. We use singular value decomposition method [12] to get the normalized singular values of quantized RSS matrix from the two datasets, which is illustrated in Fig. 2. According to these figures, the major portion of the normalized singular value of the quantized RSS matrices are smaller than 0.1 in both datasets. It concludes that the quantized RSS matrix is a low rank matrix, and the quantized RSS measurements have high correlation in a whole. Such low rank property implies that it is possible to predict the whole quantized RSS matrix by partially measuring the RSS of a few communication links.

2) Similarity: We then examine the similarity between the rows of the quantized RSS matrix. We use the Pearson product-moment correlation coefficient [13] to measure the similarity, which is defined as, given two columns X_i and X_j, the correlated coefficient between them is:

$$r_{ij} = \frac{\sum_{l=1}^{N}(X_{il} - \bar{X}_i)(X_{jl} - \bar{X}_j)}{\sqrt{\sum_{l=1}^{N}(X_{il} - \bar{X}_i)^2}\sqrt{\sum_{l=1}^{N}(X_{jl} - \bar{X}_j)^2}} \quad (2)$$

The sorted average correlation coefficients of the quantized RSS matrix for the two datasets are illustrated in Fig. 3. According to the figure, the correlations between rows are high: most of them are higher than 0.5 and in some cases they approach to 0.9. The high correlation of row values suggests that the unknown value of a row can be revealed by the value of the correlated rows with high accuracy.

The low rank and similarity properties of QCG lead to different methods to estimate the quantized RSS matrix by partial measurement, which will be introduced in the next Section.

IV. Strategies for QCG Estimation

Exhausting measurement of pair-wise RSS values could be high cost and time consuming [7], especially for large-scale outdoor networks. To achieve efficient generation of quantized conflict graph, we propose three strategies to estimate QCG based on partial measurement. The basic idea is described as follows. We first randomly choose a few links in the wireless network and measure their RSS values. Since signal propagation attenuation varies over time and environments, the measurement of each link should take several rounds to get an average value of the measurements. The number of links chosen for measurement is normally in the order of $O(N)$ (N is the size of the quantized conflict graph) and it is shown in later experiments (Fig. 4) that about 5% of measurements are enough to achieve an acceptable accuracy. Based on the partial measurement, we apply different methods to predict the unmeasured RSS values in QCG.

Before introducing the detailed estimation methods, we provide a formal definition of the QCG estimation problem.

A. Problem Definition

We consider a quantized conflict graph $G_q = (V_q, E_q, W_q)$ with size N, and assume $X = [x_{ij}]_{N\times N}$ being its corresponding quantized RSS matrix. The set of vertex pairs in G_q that are chosen for measurement is denoted by Ω. For partial measurement, $\Omega \subset E_q$ and $|\Omega| \ll |E_q|$. Assume $\hat{X} = [\hat{x}_{ij}]_{N\times N}$ is the estimated quantized RSS matrix. The QCG estimation problem is defined as:

Definition 3: ***QCG Estimation Problem****:* For a QCG represented by a quantized RSS matrix X, given a set of observed measurements $\{x_{i,j}|(i,j) \in \Omega\}$, find an estimated quantized RSS matrix $\hat{X}$, so that the different between X and $\hat{X}$ is minimized.

In the next subsections, we propose three strategies to derive the estimated quantized RSS values.

B. Estimation based on low-rank approximation

A straightforward approach is to formulate the QCG estimation problem as a matrix completion problem. In mathematics, matrix completion is the process of adding entries to a matrix that has some unknown or missing values [14], which can be applied to recover low-rank matrix. Specifically, low-rank approximation targets at solving the following problem

$$\begin{aligned} \min_{\hat{X}} \quad & rank(\hat{X}) \\ s.t. \quad & \hat{x}_{ij} = x_{ij}, \quad (i,j) \in \Omega \\ & \hat{x}_{ij} \in \mathcal{C}, \end{aligned} \quad (3)$$

where $\mathcal{C} = \{1, 2, \cdots, M\}$ is the quantized RSS levels. However, this optimization problem is NP-hard [14] and all known algorithms that provide exact solution have exponential time complexity.

To reduce computation complexity, we seek approximate solution to the QCG estimation problem. According to the above analysis, finding the $\hat{X}$ needs to fulfill several objectives: (1) to minimize the distance between $\hat{X}$ and X; (2) to minimize the rank of $\hat{X}$; (3) to retain several signal properties such as pathloss in $\hat{X}$.

The first objective requires that the estimated value $\hat{x}_{ij}$ is close to the measured value as much as possible. Since X is only partially measured, this requires minimizing the estimated error in the domain of Ω. Define $M_\Omega = [m_{ij}]_{N\times N}$ be the measured matrix over Ω where $m_{ij} = x_{ij}$ for $\forall(i,j) \in \Omega$ and $m_{ij} = 0$ for the rest. Similar, define $\hat{M}_\Omega = [\hat{m}_{ij}]_{N\times N}$ be the estimated matrix over Ω where $\hat{m}_{ij} = \hat{x}_{ij}$ for $\forall(i,j) \in \Omega$ and $\hat{m}_{ij} = 0$ for the rest. Thus objective (1) can be achieved by minimizing the Frobenius norm [14] of $M_\Omega - \hat{M}_\Omega$, which is defined by

$$||M_\Omega - \hat{M}_\Omega||_F^2 = \sqrt{\sum_{(i,j)\in\Omega} |m_{ij} - \hat{m}_{ij}|^2} \tag{4}$$

According to the study of [14], minimizing the rank of a matrix corresponds to minimizing its nuclear norm. Thus objective (2) can be achieved by minimizing the nuclear norm of $\hat{X}$ which is defined by

$$||\hat{X}||_* = \sum_i \delta_i, \tag{5}$$

where δ_i is the singular values of the matrix $\hat{X}$.

To fulfill objective (3), we apply the *Uniform Pathloss Model* [15], which is the simplest and most-used model for wireless communication. It captures signal attenuation over distance using a single pathloss exponent. In this model, RSS can be calculated by $Pd^{-\alpha}$, where P is the sending power; d is the distance of the node pair; and α is the pathloss exponent related to the environment with the typical value of 4.

The set of measurements $\{x_{i,j}|(i,j) \in \Omega\}$ can be used to calibrate this model by performing a regression. Then we get a matrix $\hat{X}_m$, whose entries, denoted as $\hat{x}_{ij}^m$ are all generated by the calibrated model. Thus, if the estimated $\hat{X}$ approximate the measured one, the ℓ_1 norm should be minimized:

$$||\hat{X} - \hat{X}_m||_1 = \sum_{i,j} |\hat{x}_{ij} - \hat{x}_{ij}^m| \tag{6}$$

Combining equations (4), (5) and (6), we have the following optimization problem for QCG estimation:

$$\begin{aligned} \min_{\hat{X}} \quad & ||M_\Omega - \hat{M}_\Omega||_F^2 + \lambda||\hat{X}||_* + \gamma||\hat{X} - \hat{X}_m||_1, \\ s.t. \quad & \hat{x}_{ij} \in \mathcal{C}, \end{aligned} \tag{7}$$

where $\lambda \in (0,1)$ and $\gamma \in (0,1)$ are the regularized coefficient balancing the observations fit, the rank of matrix $\hat{X}$, and the calibrated pathloss property.

If we use continuous $\hat{X}$, this problem is a typical convex optimization problem, so the estimated quantized RSS matrix $\hat{X}$ can be obtained by using the standard algorithms for convex optimization. The quantized version could be efficiently solved with the method introduced in [16].

Algorithm 1 WKNN anglrothm

1: Get the measurement calibrated quantized RSS matrix $\hat{X}_m$.
2: Compute the similarity between rows/columns $S(i,j)$ of matrix X'.
3: Find k nearest neighbors of $N(i,j)$ for each unknown element of $\hat{X}$.
4: Estimate the unknown elements $\hat{x}_{ij} \in \hat{X}$ using Eq. (8).
5: Quantize the predicted value $\hat{x}_{ij}$ to the range of $\mathcal{C}$.

C. Estimation based on similarity

Due to the similarity property of quantized RSS matrix, the unmeasured value in a row can be estimated by incorporating the values of other similar rows. We propose a weighted k-nearest neighbor method (WKNN) which is extended from the classical KNN algorithm [17] and exploits a weighted mean of the k-nearest neighbors values for QCG estimation.

To apply the WKNN algorithm, we use similarity as the distance measurement between two rows, which can be calculated by the Pearson product-moment correlation coefficient defined in Eq. (2).

Since the matrix X is not known in advance, we use the measurement calibrated RSS matrix $\hat{X}_m$ to estimate the mutual similarity $S(i,j)$ of row i and row j in the matrix, which is calculated by their correlation coefficient.

The set of k-nearest neighbors of the row containing x_{ij} in matrix $\hat{X}$ is denoted by $N(i,j)$. Then, the estimated value of x_{ij} is given by

$$\hat{x}_{ij} = \frac{\sum_{k\in N(i,j)}(S(k,j)x_{kj}))}{\sum_{k\in N(i,j)} S(k,j)}. \tag{8}$$

where x_{kj} is the measured or estimated values of the k-nearest neighbors. The detailed description of the algorithm is given in algorithm 1.

During the discussion, we only use the rows as the estimation reference. Noted that the proposed WKNN algorithm can also be applied to columns for matrix estimation with slight modification.

D. A comprehensive strategy

The low-rank based strategy and the similarity based strategy both have advantages and disadvantages. On one hand, low-rank approximation is a general method that can be applied to any matrix with low-rank property. But it relies on convex optimization, which always yields complicated optimization problem with high computation complexity. On the other hand, similarity based strategy only deals with the input from $k-$nearest neighbors, which is more efficient and achieves high accuracy. But it works well only when the rows are highly correlated and there are enough of measured neighbors. To make the best use of their advantages, we propose a comprehensive strategy targeting to improve the estimation accuracy.

The comprehensive strategy is described as follows.

- **Step 1:** Execute the low-rank approximation method to get a preliminary estimation denoted by $\hat{X}_{LR}$.
- **Step 2:** Compute the nearest k neighbors $N(i,j)$ of each unmeasured value $\hat{x}_{ij}$ using $\hat{X}_{LR}$.
- **Step 3:** Compute a set of weights $\{w_1, ..., w_k\}$ for each entry using the regression:

$$\hat{X}_{LR}(i,j) = \sum_{k \in N(i,j)} w_k \hat{X}_{LR}(k,j), j = \{1, 2, ..., N\}.$$

- **Step 4:** Estimate the matrix by $\hat{x}_{ij} = \sum_{k \in N(i,j)} w_k x_{kj}$, where if the x_{kj} are unmeasured entries, use the estimated value $\hat{x}_{kj}$ in $\hat{X}_{LR}$; otherwise use the measured value.

This algorithm uses the output of low-rank approximation as the input of the similarity based approach. Two changes are made to the similarity based strategy used here. First, the distance metric, which is correlation coefficients between rows, are computed using $\hat{X}_{LR}$ instead of $\hat{X}_m$. Second, the correlation coefficients between rows are not using as the weights to compute $\hat{X}$. Instead, we use linear regression to compute the best weights that combines the rows to compute $\hat{X}_{LR}(i,j)$.

E. Discussion

The accuracy of quantized RSS matrix estimation is impacted by a number of factors such the number of measurements, the levels of quantization, and spectrum diversity, which are discussed below.

1) Measurement method: The number of measurements at the wireless links directly affects the accuracy of RSS estimation. Theoretically, the more links are measured, the more accurate the estimation is. However, increasing the measurement number will also increase the cost and delay for generating the quantized conflict graph. To address this trade-off, we suggest that the links chosen from measurement is in the order of $\mathcal{O}(N)$, where N is the size of the quantized conflict graph. According to our experiments, at least 5% of entries are required to be measured to achieve a certain accuracy of RSS estimation using the proposed methods.

2) The level of quantization: Since we apply a M-level quantized conflict graph for RSS estimation, the chosen of parameter M is non-trivial. For a finer-grain partition, the quantized RSS value is more approaching to the true value. But it will harm the properties of low-rank and high correlation, which increases the difficulty of RSS prediction. For a smaller M, the prediction is more efficient and accurate, but will also deviate from the true value. The chosen of proper value of M is discussed in our experiments.

3) Spectrum diversity: We should also notice that, in the scenario of wireless network with narrowband, one conflict graph is enough to characterize the whole spectrum allocation. But for the case of wider spectrum, the effect of spectrum diversity could not be neglected. As the result, different conflict graphs should be built for different spectrum. Building quantized conflict graph over diverse spectrum efficiently will be our future work.

V. Wireless Network Optimization based on QCG

In this section, we apply the derived QCG for wireless network optimization. Specifically, we focus on the problem of interference minimization in wireless network using QCG.

We consider a wireless communication network with a set of entities $\mathcal{N} = \{1, 2, \cdots, N\}$. There are a number of wireless communication channels denoted by the set $\mathcal{S} = \{1, 2, \cdots, S\}$. The interference between pairs of the entities is described by an M-level quantized conflict graph $\mathcal{G}_q = (V_q, E_q, W_q)$. Note that if $M = 2$, G will equal to the traditional conflict graph with binary interference indication.

Interference can be avoided by assigning different communication channels to the neighboring communication nodes. One important issue in wireless network is to obtain an optimal assignment of S channels to the N communication nodes, targeting at minimizing the overall interference in the wireless network.

To make a clear statement, we introduce an indicator function $I(i,j)$ to indicate the channel assignment of individual node. For $\forall i \in \mathcal{N}$ and $\forall j \in \mathcal{S}$,

$$I(i,j) = \begin{cases} 1, & if\ node\ i\ uses\ channel\ j \\ 0. & otherwise \end{cases} \quad (9)$$

According to the definition, a QCG $\mathcal{G}_q$ can be represented by a quantized RSS matrix $Q = [q_{ij}]_{N \times N}$. Since interference occurs only when neighboring nodes working on the same channel. Thus for channel j, the total interference can be measured by the sum of RSS values over the channel: $\sum_{i \in \mathcal{N}} q_{ij} I(i,j)$. The total interference over all S channels are expressed by

$$\sum_{j \in \mathcal{S}} \sum_{i \in \mathcal{N}} q_{ij} I(i,j). \quad (10)$$

Minimizing Eq. (10) equals to maximizing the "cut" of a graph partition. Specifically, if we partition $\mathcal{G}_q$ into S groups, the problem equals to finding the best partition to maximize the total edge weights (represented by the quantized RSS value) between partitions. This is the classical Max K-cut problem [6] with $K = S$ in the case of channel assignment.

Since Max K-cut is known to be NP-hard, the interference minimization problem by channel assignment based on QCG is also NP-hard. Several approximate algorithms have been developed to solve this problem efficiently [6].

Aside from interference minimization, QCG can also be applied to optimize other objectives in wireless communication, such as minimizing network throughput, optimizing sleeping schedule in wireless networks, etc. Exploring the potential of QCG for other wireless network optimization problems will be our future work.

VI. Performance Evaluation

In this section, we evaluate the performance of the proposed algorithms. The evaluation targets are as follows.

- The accuracy of QCG estimation using the proposed low-rank based, similarity based and comprehensive strategies;
- The numerical results of the impact of system parameters;
- The impact of quantization levels;
- The performance gain of using QCG for interference minimization in wireless networks comparing to the one using traditional conflict graph;

Before presenting the numerical results, we introduce the experimental settings.

A. Experimental Settings

We use the MetroFi dataset to evaluate the proposed algorithms and optimization results. Firstly, we process the MetroFi data to generate the node-to-node RSS values and get a 70×70 matrix representing the RSS measurements. With this matrix, we now present the specific settings for different experiments.

1) Settings for quantized RSS matrix estimation: The methodology of verifying the accuracy of quantized RSS matrix estimation is to drop some entries from the measured quantized RSS matrix, and then apply the estimation strategies on them to recover the missing values. The original RSS matrix is used as ground truth for comparison.

We use the normalized Mean Absolute Error (Normalized MAE) to measure estimation accuracy, which is calculated by

$$\frac{\sum_{(i,j)\notin\Omega} |\hat{x}_{ij} - x_{ij}|}{\sum_{(i,j)\notin\Omega} |x_{ij}|} \tag{11}$$

where x_{ij} is the ground truth and $\hat{x}_{ij}$ is the estimated value.

2) Settings for QCG-based interference minimization: We examine the performance of QCG-based interference minimization and compare the result with the one based on traditional binary conflict graph.

For a fair and comprehensive comparison, we generate 100 scenarios by randomly selecting κ nodes from the MetroFi dataset to get the interference RSS measures. In our experiments, we choose κ from $\{20, 30, 40, 50, 60\}$ and randomly generate 20 scenarios for each κ.

Since different scenarios have different network size, we introduce a normalized interference reduction to evaluate the performance gain, which is defined as

$$Ratio_r = \frac{I_{network} - I_{assigned}}{I_{network}}, \tag{12}$$

where $I_{network}$ is the total network interference represented by the sum of all elements in the RSS matrix; and $I_{assigned}$ is the total network interference after applying the interference minimization algorithm.

There are several parameters used in our configuration. The default values are: $k = 8$ in the similarity based strategy; $\gamma = 0.1$ and $\lambda = 0.1$ in the low-rank based strategy; $M = 10$ for the M-level quantization. The number of available channels is assumed to be 10.

B. Numerical Result

1) Accuracy of quantized matrix estimation: We examine the accuracy of the three proposed estimation strategies, namely low-rank based method, similarity based method, and comprehensive method. The results are shown in Fig. 4. According to this figure, the comprehensive method outperforms the other methods no matter how much portion the RSS matrix's entries are measured. The similarity based method has high estimated error when the portion of measured entries is small, but it improves quickly when the portion increases, which performs as good as the low-rank method when more than 50% entries are measured. It should also be noticed that, the comprehensive solution performs only slightly better than the low-rank solution when the measured portion is low. But the performance gap between comprehensive solution and low-rank solution grows with the growing of measured portion. This proves the effectiveness of incorporate the similarity solution into comprehensive solution.

2) Sensitivity of system parameters: Sensitivity analysis of the system parameters including γ, λ, and k are shown in Fig. 5, Fig. 6 and Fig. 7 respectively. The parameter γ and λ is the regularized parameter, which adjust the portion of the norm of $||\hat{X} - \hat{X}_m||_1$ and the nuclear norm $||\hat{X}||_*$. We tune γ and λ in four discrete values, $\{10^{-3}, 10^{-2}, 0.1, 1\}$. From Fig. 5, we can see that either low-rank or comprehensive method achieve their best performance when γ is set to 0.1. Similar trends are observed for λ in Fig. 6, but the precision of low-rank and comprehensive solution is more sensitive, whose variance is larger when we vary λ. This means the nuclear norm $|\hat{X}|_*$ contributes more to the results.

The parameter k, which indicates the number of neighbors used to compute the RSS estimation, could impact the system precision. As shown in Fig. 7, when k increases, the estimated error reduces at the beginning, and increases when it reaches some level. It implies that a larger k could damage the system accuracy due to the reason that some neighbors with low correlation will bring noise to the system. The figure suggests that $k = 8$ is a proper value for quantized RSS matrix estimation.

3) Impact of the level of quantization: We are also interested in the impact of M for a M-level quantized RSS measurement. We vary M by the number $\{2, 10, 100, 1000\}$ and show the result in Fig. 8. According to the figure, the estimation error of all three algorithms improve when the quantization approaching finer grain. However, when M is larger than 10, the performance gain increases slowly, which has only marginal improvement when M varying from 10 to 1000. Since the larger M will increase the computational complexity of the algorithms, it suggests that $M = 10$ is a proper grain for quantization.

4) The performance of QCG-based interference minimization: The use of QCG to minimize wireless network interference is examined in Fig. 9. In this figure, we compare the interference reduction ratio of QCG-based optimization with the method based on traditional conflict graph (denoted by BCG in the figure) and the method based on exhausting

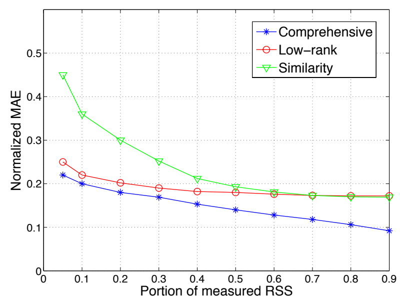


Fig. 4: Prediction accuracy comparison for three proposed methods.

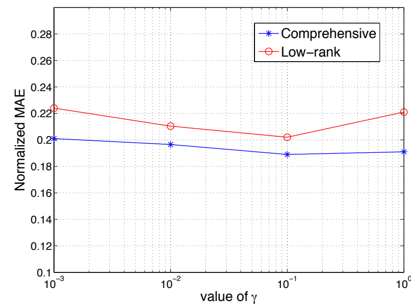


Fig. 5: The sensitivity with the regulized parameter γ.

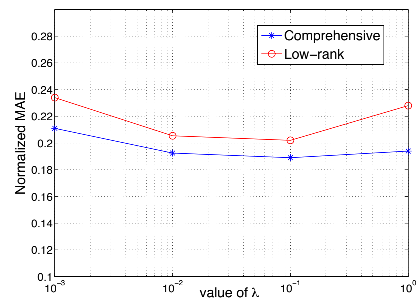


Fig. 6: The sensitivity with the regulized parameter λ.

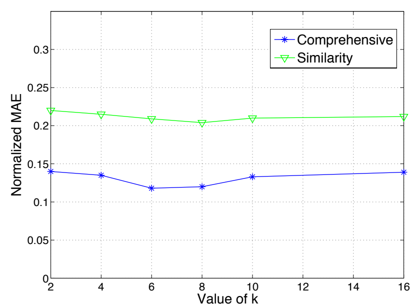


Fig. 7: The sensitvity with the parameter k in similarity method.

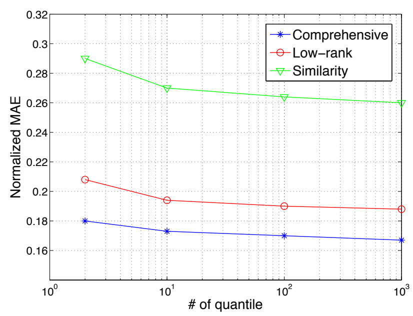


Fig. 8: The impact of quantized number to the Normalized MAE.

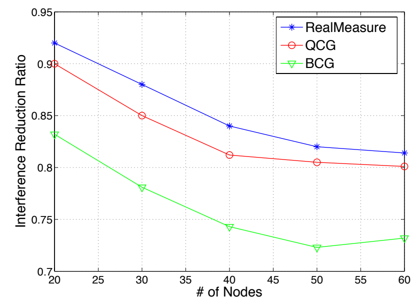


Fig. 9: The comparison of network interference minimization using QCG and binary CG.

RSS measurement (denoted by RealMeasure). As shown in the figure, QCG performs much better than BCG, which has the interference reduction ratio gap larger than 12%. The result of "RealMeasure" is the theoretical upper bound of interference reduction, and the figure shows that QCG approaches to this upper bound with the gap less than 6%.

VII. Related Works

In this section, we survey the related work of our paper in three areas, namely, the work in interference model, the work in wireless network optimization, and the ones in area of matrix completion.

A. Interference model and Measurement Calibration

Many research works are endeavoured to bring more accurate interference models. Theoretically, in [3], both conflict graph and SINR model are proposed. For practical reasons, in [18], Qiu and et al. proposed an generic interference model using per-link signal measurements to capture interference conditions among individual links. Recently, Zhou and et al. [7] studied how to practically generate accurate conflict graph for outdoor wireless network using measurement calibrated signal propagation model. Aside from conflict graphs, recent work examines the accuracy of general interference models for small-scale networks using per-link measurements [19].

Our work differs from these work by exploring the low rank property and locality of RSS matrix and introduce set of matrix completion based efficient RSS matrix estimation methods.

B. The wireless network optimization based on interference model

The interference models are developed to ease the modelling and optimization of wireless networks. The milestone work [3] proposed several famous theoretical results of the capacity of wireless networks based on both protocol interference model (conflict graph) and physical interference model (SINR model). The conflict graph, considering the spectrum diversity property, had been extended to multi-dimensional conflict graph (MDCG), for general MR-MC networks [5], [20]. Recently, in [4], Joo and et al. proposed a distributed approximation algorithm for MWIS of link scheduling in wireless network based on conflict graph. The SINR model is frequently adopted in cross layer optimization of wireless networks. Due to its computational complexity, [21], [22] followed the layer-decoupled approach for analysis, under which, the solution was obtained by determining an algorithm/mechanism for one layer at a time, and then, piecing up them together without the need of solving a joint optimization problem. In [23], Bhatia and Kodialam optimized power control and routing, but assumed some frequency hopping mechanism was in place for scheduling, which simplified the joint consideration of scheduling. Shi and et al. [24] modelled the cross layer throughput optimization problem as MINLP problem and solve it using branch and bound framework.

C. Matrix interpolation and its application in networks

The fundamental theory works of matrix interpolation of low-rank matrix was brought by [14], [25]. This method is universally applied in the area, where the data matrix have low-rank property and missing values. In network research area, the matrix interpolation and its vector version compressive sensing are applied to many problems. For example, compressive sensing was introduced to solve the traffic matrix derivation and interpolation problem [26]. In [27] interpolation of low rank matrix is also used for localization problem. In [9], it was applied for efficient SINR estimation problem. To tackle the matrix with discrete elements, both centralized [16] and distributed [28] solutions were proposed.

VIII. Conclusion

In this paper, we propose the model of quantized conflict graph, which generalized the 0-1 measurement of traditional conflict graph binary to a M-level quantization. The benefits of QCG lie on twofold: it provides a finer-grain measurement of interference condition and can deal with the accumulative effect of multiple small amount interference. We explore the properties of QCG, such as low-rank matrix and high-similarity between the interference measurements by analysing two datasets collected from indoor and outdoor environments. To achieve efficient construction of QCG, we propose three QCG estimation strategies called low-rank approximation approach, similarity based approach, and comprehensive approach. We further applied QCG for wireless network optimization by studying the QCG-based network interference minimization problem, which can be solved by tackling the classical Max K-cut problem. We also conduct extensive experiments using real wireless network datasets to evaluate the performance of the proposed algorithms, which show that the proposed QCG estimation strategies achieve low estimation error for proper system settings, and approach the upper bound of performance gain for interference minimization in wireless network.

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