Selecting Optimal Mobile Users for Long-term Environmental Monitoring by Crowdsourcing

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ABSTRACT

Urban environmental monitoring related to such issues as air pollution and noise helps people understand their living environments and promotes urban construction. It is more and more important nowadays. By crowdsourcing, we can get mobile users at a low cost to collect measurement at different locations. This paper studies how to select optimal mobile users to construct an accurate monitoring map under a limited budget. We extend the noise Gaussian Process model to construct the data utility model. Because the monitoring map is updated in each time slot, we try to maximize the time-averaged data utility under the time-averaged budget constraint. This problem is particularly challenging given the unknown future information and the difficulty of solving the one-slot problem: maximizing a non-monotone submodular objective under the budget constraint. To address these challenges, we first make use of Lyapunov optimization to decompose the long-term optimization problem into a series of real-time problems which do not require a priori knowledge about the future information. We then propose a time-efficient online algorithm to solve the NP-hard one-slot problem. As long as the algorithm for the one-slot problem has a competitive ratio e, the time-averaged data utility of our online algorithm has a small gap compared with e times the optimal one. Evaluations based on the real air pollution data in Beijing [2] and real human trajectory data [1] show the efficiency of our approach.

CCS CONCEPTS

• **Human-centered computing** → *Collaborative and social computing design and evaluation methods.*

KEYWORDS

Environmental monitoring, Crowdsourcing, Gaussian process, Long-term problem, Non-monotone submodular function

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1 INTRODUCTION

Urban environmental monitoring on issues such as air pollution [4][13], noise [3][29], solar [20] and wireless signal [11][26] is more and more important nowadays. Environmental monitoring helps people understand their living environments and promotes urban construction such as pollution abatement and wireless network development. Existing environmental monitoring applications on industry[4][3] use pre-deployed sensor networks to collect monitoring data. Because of a high deployment and maintenance cost, the sensing node distribution is sparse and monitoring is coarse-grained. Instead, we use crowdsourcing over mobile devices which are embedded with rich sensors [8][5]. Crowdsourcing is cheap and can provide fine-grained monitoring because mobile devices such as smartphones are ubiquitous.

As shown in Fig. 1, in environmental monitoring by crowdsourcing, such as air pollution monitoring, the considered area, such as a city, is divided into many grids. The color of a grid is darker if pollution is serious and lighter in the contrary case. The monitoring map is updated in each time slot. In a time slot, the crowdsourcer recruits mobile users to collect measurements. Because of budget limitation, some grids do not have measurements. The crowdsourcer needs to infer levels of air pollution in these grids.

The big concern of the crowdsourcer is: how to get accurate monitoring maps under the limited average budget. As shown in Fig. 2, the total payment in a slot can be dynamic as long as the average payment is below the set budget B_{avg}. What's more, because the available budget in a slot is finite, the total payment in a slot cannot exceed the upper bound U. We do not set a fixed budget in a slot because the situation in each slot changes dynamically. In some slots, the number of mobile users is small, costs of mobile users are high or qualities of mobile users are low. Spending too much budget on these slots is useless. We should put more budget in other slots to achieve higher average accuracy of monitoring maps.

The problem is non-trivial mainly due to the following challenges. *First*, the time-averaged objective and budget

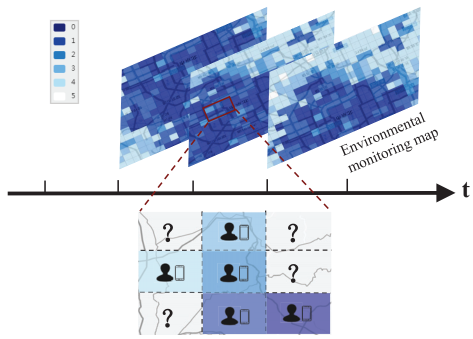

Figure 1: Environmental monitoring by crowdsourcing.

constraint couple the short-term mobile user selection problem across time. The dynamic future information, including locations, costs and qualities of mobile users, is not available in advance. Thus the decisions have to be made without foreseeing the future. *Second*, even if we only consider one slot, the short-term problem is NP-hard whose optimal solution cannot be found in polynomial time. *Third*, to decouple the lone-term problem, we need to adjust the original short-term problem. The adjusted short-term problem is even harder than the original one. It is a non-monotone submodular optimization under the knapsack constraint. The time complexities of its approximate algorithms are all extremely high [16][7]. For example, in the work of Fadaei et al. [7], when the approximation ratio is set to 0.05, the algorithm is just an exhaustive algorithm if the number of all mobile users is less than 10000.

To overcome the above challenges, we propose an online algorithm to select mobile users in each slot. We first adopt the noise-aware Gaussian Process (GP) model for sensing data and extend it to construct our data utility model. GP is usually used to estimate the distribution of unobserved random variables by observed values of related random variables. We use the entropy of the estimated distribution as the uncertainty of estimation. If we are more confident about the estimation, the data utility of measurements is large. Then we formulate our time-averaged data utility maximization problem under the limitation of the average budget and the upper bound β of the budget in a slot.

We apply the Lyapunov optimization technique to transform this long-term problem to a series of real-time one-slot optimization problems. Each one-slot problem selects mobile users to maximize a non-monotone submodular objective under the limited budget β. This objective considers both the data utility and the overused budget. We propose an online algorithm to solve the one-slot problem. The online algorithm does not pick any mobile user from the first half of the stream. At the end of the first half, it computes the highest objective G achieved by mobile users in the first half and uses $\rho = G/\beta$ as an reference of the contribution per cost of mobile users which will be selected in the second half of stream. In the second half, the crowdsourcer selects a mobile

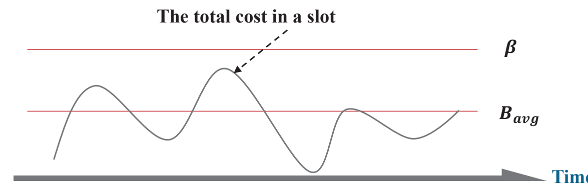

Figure 2: The budget constraint.

user if the remaining budget can afford the cost, and moreover, the contribution per cost is at least ρ. The difficulty is how to compute G. As we introduce above, approximation algorithms for the one-slot problem usually have high time complexities. Therefore, we recursively call this online algorithm until the number of mobile users is very small and we can easily get the solution by enumerating all possible solutions. We have made the following main technical contributions.

- We show that as long as the algorithm for the one-slot problem has a competitive ratio e, the time-averaged data utility of our online algorithm has a small gap compared with e times the optimal one.
- We prove that the one-slot problem is a non-monotone submodular optimization under the knapsack constraint and propose an online algorithm with computing efficiency to solve the one-slot problem.
- We evaluate our approach using the real dataset and compare our approach with other baselines to show its efficiency.

Paper organization. The remainder of this paper is structured as follows. The system model and problem formulation are presented in Section 2. In Section 3, we describe the design details of the online algorithm. We present the theoretical analysis in Section 4. We evaluate the performance of the proposed algorithm in Section 5. Related work is discussed in Section 6. Finally, we conclude in Section 7.

2 MODELING AND PROBLEM FORMULATION

2.1 System Model

The environmental monitoring map needs to be updated at each time slot $t \in [0, \cdots, T-1]$. The set of grids on the map is $L = \{l_1, \cdots, l_m\}$. In each slot t, a set of mobile users $U^t = \{u_1^t, u_2^t, \cdots, u_{n_t}^t\}$ is willing to provide sensing services. The notation n_t is the number of mobile users in slot t. The location of mobile user u_i^t belongs to grid $\hat{l}_i^t$ and the cost is c_i^t. A mobile user u_i^t has noise n_i^t compared with the truth. The noise can be estimated from the collected historical data [18]. The average total cost of recruited mobile users in a slot cannot exceed the average budget B_{avg}. The total cost in a slot has an upper bound β. The interactions between the crowdsourcer and mobile users are as follows. The length of a time slot is $\mathcal{T}$. In a slot, mobile users come online. When a mobile user arrives, he/she submits the information including the location and the cost. Then the crowdsourcer determines whether to recruit this user or not immediately because this mobile user may move to another grid later. Selected mobile users collect measurements in corresponding grids and upload them to the crowdsourcer. After receiving all uploaded data at the end of the time slot, the crowdsourcer pays mobile users according to their costs. Then environmental values of observed grids are calculated and environmental values of unobserved grids are estimated from collected measurements. Because of the limited budget, the crowdsourcer chooses at most one mobile user for each grid.

2.2 Data Utility Model

Obviously, the crowdsourcer hopes that measurements at important grids can be collected. Then exact environmental values of these grids can be obtained by filtering noises [18]. At the same time, inferences of other grids should be as accurate as possible. To select a set of mobile users that satisfy the requirement of the crowdsourcer, we use data utility to model the contribution of measurements uploaded by a set of mobile users. In the data utility model, the importance of grid l_i is denoted as e_i. Intuitively, a grid with a higher population density is more important because more people refer to the environmental value at this grid. The accuracy of inferences is usually measured by two metrics: Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) [29][11]. Assume that $\hat{x}_i^t$ is an inference of grid l_i in slot t and x_i^t is the ground truth. The two metrics are defined as follows.

$$\begin{aligned} RMSE^t &= \sqrt{\frac{\sum_{l_i}(x_i^t - \hat{x}_i^t)^2}{m}} \\ MAE^t &= \frac{\sum_{l_i}|x_i^t - \hat{x}_i^t|}{m} \end{aligned} \tag{1}$$

To achieve higher accuracy, the crowdsourcer should select mobile users who can reduce the error to the minimum value. However, it is difficult to calculate the error before mobile user selection, which is a common problem for environmental map construction [29][11]. To solve this problem, we adopt the noise-aware Gaussian Process model [9] for sensing data, and extend it to involve certainty of estimated environmental values.

In GP, the environmental value X_i^t of grid l_i in slot t is assumed to be a random variable which follows a one-dimension Gaussian distribution with mean μ_i^t and variance Σ_i^t. The joint distribution over environmental values in all grids is a multivariate normal distribution with a mean vector $\mu^t = \{\mu_1^t, \cdots, \mu_m^t\}$ and a symmetric positive-definite covariance matrix $\mathbf{\Sigma}^t$ whose element $\Sigma_{i,j}^t$ is the covariance for environmental values in grids l_i and l_j. The value of $\Sigma_{i,i}^t$ is equal to Σ_i^t.

At the beginning of each time slot, we have an initial mean vector and an initial covariance matrix. They can be estimated as average values of means and covariances in the recent past or means and covariances in the same time duration in the previous cycle. Initial covariances of the first time slot can be estimated through distances as shown in the evaluation. The longer the distance between two grids is, the larger the covariance is. As for means, before the first time slot, we can recruit more mobile users to measure environmental values in all grids and use them as initial means. This is the necessary cost of starting up the environmental monitoring project.

We next introduce how to infer environmental values of unobserved grids after receiving measurements from mobile users. Suppose the set of grids having measurements in slot t is $A_t \subseteq L$ and the unobserved grid set is $R_t = L \backslash A_t$. The mean vector and the covariance matrix of the joint distribution of environmental values can be expressed by the following form, where μ_{A_t} and μ_{R_t} are vectors and $\mathbf{\Sigma}_{A_tA_t}$, $\mathbf{\Sigma}_{A_tR_t}$, $\mathbf{\Sigma}_{R_tA_t}$ and $\mathbf{\Sigma}_{R_tR_t}$ are matrices.

$$\mu^t = \begin{bmatrix} \mu_{A_t} \\ \mu_{R_t} \end{bmatrix} \quad \mathbf{\Sigma}^t = \begin{bmatrix} \mathbf{\Sigma}_{A_tA_t} & \mathbf{\Sigma}_{A_tR_t}, \\ \mathbf{\Sigma}_{R_tA_t} & \mathbf{\Sigma}_{R_tR_t}. \end{bmatrix} \tag{2}$$

In the noise matrix Γ_{A_t}, the diagonal entries are noises of mobile users collecting measurements in corresponding grids in set A_t and the other entries are zeros. Then given the measurement vector x_{A_t}, we can calculate the probability distribution $P(X_R|X_{A_t} = x_{A_t})$, which is a conditional Gaussian distribution with the mean vector $\mu_{R_t|A_t}$ and the variance matrix $\Sigma_{R_t|A_t}$. The environmental values of unobserved grids are inferred as the means in $\mu_{R_t|A_t}$.

$$\begin{cases} \mu_{R_t|A_t} = \mu_{R_t} + \Sigma_{R_tA_t}(\Sigma_{A_tA_t} + \Gamma_{A_t})^{-1}(x_{A_t} - \mu_{A_t}) \\ \Sigma_{R_t|A_t} = \Sigma_{R_tR_t} - \Sigma_{R_tA_t}(\Sigma_{A_tA_t} + \Gamma_{A_t})^{-1}\Sigma_{A_tR_t} \end{cases} \tag{3}$$

The entropy of the random Gaussian variable X_{R_t} is a good measurement of the uncertainty of our inference. If its distribution is very smooth and goes to the uniform distribution, the uncertainty of the inference and the entropy are both high. If its distribution is steep and the probability of the mean is close to 1, we would be very confident about our inference and the entropy is low. Therefore, the contribution of x_{A_t} to uncertainty reduction can be set as the difference between the entropy $H(X_{R_t})$ of X_{R_t}, and the entropy $H(X_{R_t}|X_{A_t})$ of X_{R_t} on condition that X_{A_t} has been observed. According to the definition of entropy, $H(X_{R_t})$ and $H(X_{R_t}|X_{A_t})$ can be calculated as follows.

$$\begin{aligned} H(X_{R_t}) &= \frac{1}{2}ln[(2\pi e)^{|R_t|}|\Sigma_{R_tR_t}|] \\ H(X_{R_t}|X_{A_t}) &= \frac{1}{2}ln[(2\pi e)^{|R_t|}|\Sigma_{R_t|A_t}|] \end{aligned} \tag{4}$$

For the selected set of mobile users S_t in slot t, the contribution $F(S_t)$ of their measurements considers both the importance of observed grids and the certainty of estimated environmental values inferred by the uploaded measurements. The set of grids having measurements is $A_t = \{\hat{l}_i^t | u_i^t \in S_t\}$ and the set of unobserved grids is $R_t = L \backslash A_t$. Then $F(S_t)$ can be expressed by the following formula.

$$F(S_t) = \sum\nolimits_{l_i \in A_t} e_{l_i} + W[H(X_{R_t}) - H(X_{R_t}|X_{A_t})]. \tag{5}$$

The notation W is the tradeoff between the importance of observed grids and the certainty of estimation.

2.3 Time-averaged Data Utility Maximization Problem

Because the crowdsourcer needs to monitor environmental values for a long time period, we propose to optimize the long-term time-averaged data utility subject to the time-averaged budget constraint, which is formulated as follows.

$$\begin{aligned} max \quad & \frac{1}{T}\lim_{T\to\infty}\sum_{t=1}^{T} F(S_t) \\ s.t. \quad & A_t = \{\hat{l}_i^t | u_i^t \in S_t\},\ \forall t \\ & R_t = L \backslash A_t,\ \forall t \\ & S_t \subseteq U^t,\ \forall t \\ & F(S_t) = \sum_{l_i \in A_t} e_{l_i} + H(X_{R_t}) - H(X_{R_t}|X_{A_t}),\ \forall t \\ & C_t = \sum_{u_i^t \in S_t} c_i^t,\ \forall t \\ & \frac{1}{T}\lim_{T\to\infty}\sum_{t=1}^{T} C_t \le B_{avg} \\ & C_t \le \beta,\ \forall t. \end{aligned} \tag{6}$$

The sixth constraint means that the average total cost of recruited mobile users cannot exceed the average budget B_{avg}. The last constraint restricts the upper bound of the total cost in a time slot. We assume that in each grid, there is only one mobile user. In this setting, the problem has been NP-hard and very difficult to solve. The scenario where there are multiple mobile users in a grid will be considered in the future work.

3 MOBILE USER SELECTION ALGORITHM

In this section, we present our mobile user selection algorithm. In the time-averaged data utility maximization problem, mobile user selection in different slots are coupled with each other. Decision making is difficult because the future information is not available in advance. We first exploit Lyapunov optimization [19] to transform the long-term problem to a series of real-time one-slot optimizations. We then present details of our online algorithm.

3.1 Decoupling the Time-averaged Problem

To decouple the time-averaged problem, we first construct a queue to represent the over spent budget. Then we show that as long as the queue is stable, the time-averaged budget constraint would be satisfied. At last, we formulate the real-time one-slot problem which maximizes the current data utility and controls the length of the queue at the same time. The real-time problems in all slots compose the original long-term problem.

3.1.1 Queue construction. We construct a virtual queue Q whose length at slot t, $Q(t)$, represents the over used budget at the beginning of slot t. We assume that the initial queue backlog is 0, i.e., $Q(0) = 0$. The queue length is updated by the following formula.

$$Q(t+1) = max[Q(t) + C_t - B_{avg}, 0]. \tag{7}$$

3.1.2 Queue stability. We show that as long as the virtual queue Q is stable, i.e., $\lim_{T\to\infty} \mathbb{E}\{Q(T)\}/T = 0$, the time-averaged budget constraint is satisfied. By adding up the inequality $Q(t+1) \ge Q(t) + C_t - B_{avg}$ derived from equation (7) over time, we get

$$\frac{Q(T)-Q(0)}{T} + B_{avg} \ge \frac{1}{T}\sum_{t=0}^{T-1} C_t. \tag{8}$$

Because $Q_i(0) = 0$, we can take an expectation of the above inequality and have

$$\lim_{T\to\infty} \frac{\mathbb{E}(Q(T))}{T} + B_{avg} \ge \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{C_t\}.$$

If the virtual queue Q is stable, we can get

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{C_t\} \le B_{avg}.$$

That is to say, the long-term time-averaged budget constraint is satisfied.

To stabilize the virtual queue, we define a quadratic Lyapunov function as follows.

$$L(Q(t)) = \frac{1}{2}Q^2(t), \tag{9}$$

The Lyapunov function represents a measure of queue congestion. Intuitively, to achieve queue stability, we should control growth of the Lyapunov function, which is measured by the one-step conditional Lyapunov drift $\Delta(Q(t))$.

$$\Delta(Q(t)) = \mathbb{E}\{L(Q(t+1)) - L(Q(t))|Q(t)\}. \tag{10}$$

The above conditional expectation is with respect to randomness in the submitted information of mobile users. The drift $\Delta(Q(t))$ denotes the change of the Lyapunov function over one slot.

3.1.3 One-slot utility-minus-drift maximization. In order to consider queue stability and data utility at the same time, we define a utility-minus-drift function as follows.

$$VF(S(t)) - \Delta(Q(t)). \tag{11}$$

The non-negative control parameter V represents how much we stress data utility.

This objective is hard to optimize because of its quadratic terms. Instead, we optimize its lower bound given by Theorem 3.1. The proof is shown in Appendix A.

THEOREM 3.1. *For any slot t, for any possible $Q(t)$ by using any mobile user selection algorithm, the utility-minus-drift function could be bounded as follows.*

$$\begin{aligned} & VF(S(t)) - \Delta(Q(t)) \ge \\ & VF(S(t)) - Q(t)\mathbb{E}\{C_t - B_{avg}|Q(t)\} - D, \end{aligned}$$

where $D = \frac{1}{2}(\beta^2 + B_{avg}^2)$ is a constant value.

As shown in Theorem 4.1, by optimizing the lower bound, we can get a pretty good performance with respect to the time-averaged data utility and the queue would be stable. At the beginning of each time slot, $Q(t)$ is observed, thus $Q(t)B_{avg}$ is a constant. We just need to maximize $VF(S(t))-$

Algorithm 1: Online Mobile User Selection Algorithm

Require: L, $\{e_{l_i}|l_i \in L\}$, V, B_{avg}, β, initial μ and $\mathbf{\Sigma}$

$Q(0) = 0$.
for $t = 1, \cdots, T$ **do**
  Solve the one-slot problem, i.e., problem (12) by Alg. 2.
  Update the lenght of the virtual queue according to Equ. 7.

$Q(t)C_t$ in each slot. Then the one-slot problem in slot t, i.e., the utility-minus-drift maximization problem, is defined as

$$\begin{aligned} &max\ VF(S_t) - Q(t)C_t \\ &s.t.\ \ A_t = \{\hat{l}_i^t | u_i^t \in S_t\} \\ &\quad\ \ R_t = L \backslash A_t \\ &\quad\ \ S_t \subseteq U^t \\ &\quad\ \ F(S_t) = \textstyle\sum_{l_i \in A_t} e_{l_i} + H(X_{R_t}) - H(X_{R_t}|X_{A_t}) \\ &\quad\ \ C_t = \textstyle\sum_{u_i^t \in S_t} c_i^t \\ &\quad\ \ C_t \leq \beta. \end{aligned} \tag{12}$$

This problem is NP-hard. It is the classic knapsack problem if the objective is additive. We prove that the objective is a submodular function which is defined by the following definition. As submodularity includes additivity, this problem is harder than the classic knapsack problem.

*Definition 3.2 (**Submodular function**).* For a finite universe set U, $S_1 \subseteq S_2 \subseteq U$ and $u \in U \backslash S_2$. Then, a function G is a submodular function if and only if $G(S_1 \cup \{u\}) - G(S_1) \geq G(S_2 \cup \{u\}) - G(S_2)$.

THEOREM 3.3. *The objective $G(S_t) = VF(S_t) - Q(t)C_t$ is a non-monotone submodular function.*

We cannot directly adopt classical online and offline greedy algorithms for monotone submodular optimization [27][12], because the objective $G(S_t)$ is a non-monotone submodular function. If it is a monotone submodular function, $G_{u_i^t}(S_t) = G(S_t \cup \{u_i^t\}) - G(S_t)$ must be equal to or larger than 0 for any subset $S_t \in U^t$ and any mobile user $u_i^t \in U^t \backslash S_t$ [6]. However, $G_{u_i^t}(S_t)$ might be negative if the growth of $VF(S_t)$ is small or the growth of $Q(t)C_t$ is large.

3.2 Online Mobile User Selection Algorithm

We design an online mobile user selection algorithm shown in Alg. 1. In each slot, the crowdsourcer solves the one-slot problem (12) online by Alg. 2. At last, the length of the virtual queue is updated according to Equ. 7. Alg. 1 is online, which means that making decisions in the current slot does not need the information of the future slots. Alg. 2 is also online, which means that determining whether to recruit the coming mobile user in the current time step in a slot does not need to know mobile users coming latter in this slot.

To explain the online algorithm solving the one-slot problem, we first define marginal efficiency of a mobile user which is the marginal contribution per cost with respect to the current set of selected mobile users. More formally, it is defined as follows.

*Definition 3.4 (**Marginal Efficiency**).* Given the current set of selected mobile users in slot t, denoted by S_t, the marginal contribution of a newly coming mobile user u_i^t is

$$G_{u_i^t}(S_t) = G(S_t \cup \{u_i^t\}) - G(S_t). \tag{13}$$

Then, marginal efficiency of mobile user u_i is the marginal contribution per cost, i.e., $G_{u_i^t}(S_t)/c_i^t$.

The algorithm for the one-slot problem is inspired by the work of Bateni et al. [6]. In a slot with length $\mathcal{T}$, mobile users come online. The algorithm in this work [6] does not pick any mobile user from the first half of the stream. At time step $\mathcal{T}/2$, it finds a set S' of mobile users from the first half to achieve the highest objective with respect to the knapsack constraint, i.e., the total cost cannot exceed β. Then, the crowdsourcer obtains an estimation $G(S')$ of the optimal objective by looking at the first half of the stream and gets a threshold $\rho = G(S')/(6\beta)$ of marginal efficiency. The factor 6 is added because of submodularity. Mobile users who come later have lower marginal efficiency. If the threshold is too high, we may not recruit enough mobile users. In the second half, the crowdsourcer selects a mobile user if the remaining budget can afford the cost, and moreover, the marginal efficiency is at least ρ.

Algorithm 2: Online Algorithm for One-slot Problem

Require: L, $\{e^t_{l_i}|l_i \in L\}$, V, β, μ^t, $\mathbf{\Sigma}^t$, $Q(t)$ and $\mathcal{T}$.
Ensure: x_{A_t} and inference μ_{R_t}.

$(\tau, j, S^t) \leftarrow (0, 16, \emptyset)$
while $\tau \leq 8$ **do**
  if *there is a mobile user* u_i^t *arriving at time step* τ **then**
    $S^t = S^t \cup \{u_i^t\}$
  if $\tau = 8$ **then**
    $\rho = G(S_t)/(6 \cdot \beta)$
    $S_t = \emptyset$
  $\tau = \tau + 1$
while $8 < \tau \leq \mathcal{T}/2$ **do**
  if *there is a mobile user* u_i^t *arriving at time step* τ **then**
    if $\rho \leq \frac{G_{u_i^t}(S_t)}{c_i^t}$ && $c_i^t \leq \beta - \sum_{u_i^t \in S_t} c_i^t$ **then**
      $S_t = S_t \cup \{u_i^t\}$
    if $\tau = j$ **then**
      $\rho = G(S_t)/(6 \cdot \beta)$
      $S_t = \emptyset$
      $j = 2j$
  $\tau = \tau + 1$
while $\mathcal{T}/2 < \tau \leq \mathcal{T}$ **do**
  if *there is a mobile user* u_i^t *arriving at time step* τ **then**
    if $\rho \leq \frac{G_{u_i^t}(S_t)}{c_i^t}$ && $c_i^t \leq \beta - \sum_{u_i^t \in S_t} c_i^t$ **then**
      $S_t = S_t \cup \{u_i^t\}$
      Collect measurement x_i^t.
  $\tau = \tau + 1$
Calculate μ_{R_t}.

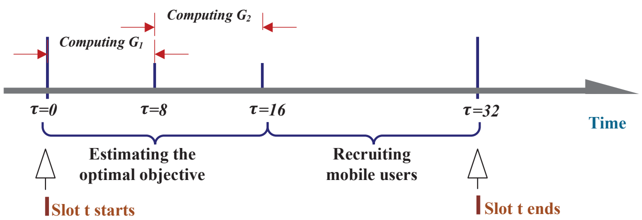

Figure 3: The online algorithm for the one-slot problem with multiple stages in a slot. In this example, the time length is 32. There are 3 stages.

The difficulty is how to get the set S'. The work of Bateni et al. does not give the solution. The one-slot problem is NP-hard. Thus, the optimal solution cannot be found in polynomial time. There are several offline approximate algorithms for non-monotone submodular functions [16][7]. However, their time complexities are also very high. We take the newest algorithm as an example [7]. This algorithm has an approximation ratio $0.25 - 2\varepsilon$ where the constant ε is positive and smaller than 1. It enumerates all mobile user sets whose number of elements is less than $1/\varepsilon^4$. For each set, it adds some other mobile users to form a candidate solution. At last, it chooses the best one from all candidate solutions. If ε is 0.1, the approximation ratio is 0.05 which is not good for an offline algorithm. What's more, if the number of all mobile users is less than $1/0.1^4 = 10000$, the algorithm is just an exhaustive algorithm. The time complexity can be up to 2^{10000}.

To overcome the high complexity of getting S', we propose our algorithm shown in Alg. 2. From $\mathcal{T}/2$, we recursively call the previously introduced online algorithm until the number of mobile users is 8. Under the following setting, if there are only 8 mobile users, the optimal solution is recruiting them all. In the setting, we assume that the environmental monitoring map has many grids and needs many mobile users. Therefore, the budget upper bound β is large and can always afford the cost of 8 mobile users. Besides, we would set V to an appropriate value so that if the number of recruited mobile users is less than 8, the marginal contribution of a new mobile user is positive.

Specifically, the time length $\mathcal{T}$ in a slot is divided into $h=\lfloor \log_2 \mathcal{T} \rfloor - 2$ stages as shown in Fig. 3. The k-th stage ends at time-slot $\mathcal{T}_k = \lfloor \mathcal{T}/2^{h-k} \rfloor$. At the end of the first stage, i.e., time step 8, we get G_1 which is the objective achieved by all arrived mobile users. We use $\rho = G_1/(6\beta)$ as the threshold to choose a mobile user set under budget β in the second stage (from time step 8 to time step 16) to get G_2. Similarly, in each stage k, we compute G_k according to the threshold $\rho = G_{k-1}/(6\beta)$ under budget β. At the end of stage $h-1$, we can get G_{h-1} and the threshold $G_{h-1}/(6\beta)$. In the last stage, we recruit a mobile user if the marginal efficiency is not less than the threshold and the remaining budget can afford the cost.

We can see that the time complexity of the algorithm is $O(n_t)$ where n_t is the number of mobile users coming in time slot t. Under the linear time complexity, the algorithm can achieve at least 0.7 times the optimal objective as shown in evaluations.

4 THEORETICAL ANALYSIS

In this section, we analyze theoretically the performance of our online algorithm. The proofs are shown in Appendix. Through Theorem 4.1, we show that although the solution of the one-slot problem is not optimal, we can still get a bound on the time-averaged data utility.

THEOREM 4.1. *Assume C_t is i.i.d. over time slots, F^{opt} is the optimal time-averaged data utility with the overall information and the one-slot algorithm has a competitive ratio e. For any non-negative control parameter V, the long-term data utility implemented by our algorithm satisfies:*

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{F(S_t)\} \geq eF^{opt} - \frac{D}{V}. \quad (14)$$

Theorem 4.2 studies the time-averaged length of the virtual queue.

THEOREM 4.2. *Assuming that $B_{avg} > 0$ and $Q(0) = 0$, then the time-averaged length of the virtual queue has the following upper bound.*

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q(t)\} \leq \frac{(1-e)VF^{opt} + B_{avg}}{e\gamma}. \quad (15)$$

The above formula means that $\lim_{T\to\infty} \mathbb{E}\{Q(T)\}/T = 0$ for $\forall m_i \in \mathbb{M}$.

Combing Theorem 4.1 and Theorem 4.2, we can see that the parameter V controls the tradeoff between the data utility and the over consumed budget. When the value of V is small, the upper bound of the time-averaged queue length is small but the gap between the time-averaged data utility and e times the optimal data utility is large. When the value of V is large, the opposite was the case.

5 EVALUATION

Based on the real air pollution data in Beijing [2] and real human trajectory data [1], we compare our algorithm with baselines to verify the theoretical results and show the efficiency of our algorithm.

5.1 Experiment Settings

We consider an application of air pollution monitoring. The default settings are as follows. The number of slots is 2800 and the length of one slot is $1h$. We set V, W, β and B_{avg} to 10, 100, 700\$ and 450\$, respectively. We divide Beijing into $2km{\cdot}2km$ grids. The importance of each grid is in $\{1, 2, 3, 4, 5\}$. The grid close to commercial centers or residential areas has a high importance because it has dense population.

The number of mobile users is 2000. We set the locations of each mobile user according to the real human trajectory data [1] which consists of 3-year trajectories of 182 users. We use trajectories of a user in different time durations to simulate multiple users. The cost of a mobile user u_i in different slots follows a truncated normal distribution $\mathcal{N}(\delta_i, v_i, lb_i, ub_i)$ with the mean δ_i, the variance v_i, the lower bound lb_i and

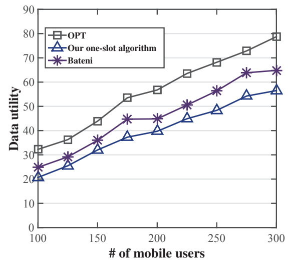

Figure 4: Data utility vs. number of mobile users.

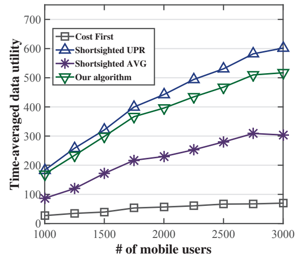

Figure 5: Time-averaged data utility vs. number of mobile users.

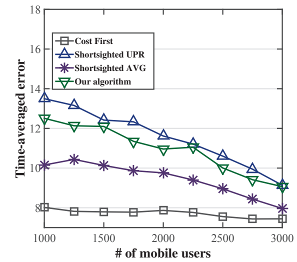

Figure 6: Time-averaged error vs. number of mobile users.

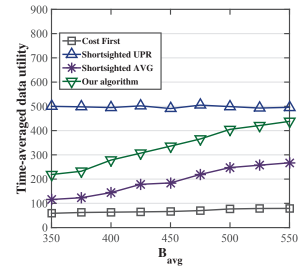

Figure 7: Time-averaged data utility vs. average budget.

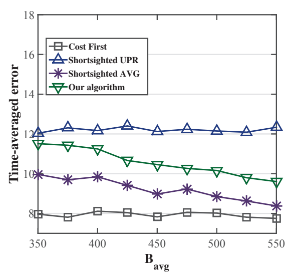

Figure 8: Time-averaged error vs. average budget.

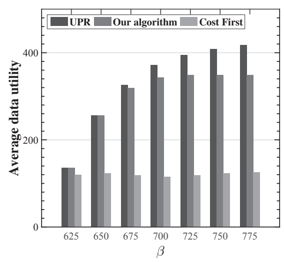

Figure 9: Time-averaged data utility vs. upper bound β of the budget.

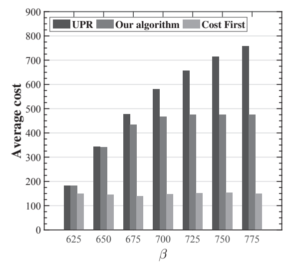

Figure 10: Time-averaged cost vs. upper bound β of the budget.

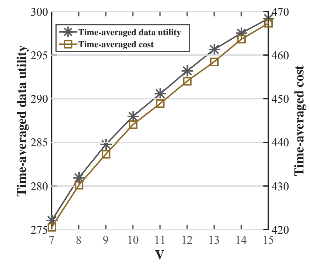

Figure 11: Time-averaged data utility and time-averaged cost vs. V.

the upper bound ub_i. The lower bound of the cost of a mobile user is selected from the uniform distribution from 0.2\$ to 0.5\$. The upper bound is generated by the uniform distribution from the corresponding lower bound to 1.5\$. The mean δ_i of the cost of a mobile user is $\frac{lb_i+ub_i}{2}$. The variance v_i is $0.2 \cdot (\delta_i - lb_i)$. We update the noise of a mobile user in each slot according to the work of Liu et al. which maximizes likelihood by EM algorithm [18].

At the beginning of each time slot, we have an initial mean vector and an initial covariance matrix. The mean (covariance) of a grid is estimated as the weighted sum of the average value of its mean (covariance) in the recent past and the mean (covariance) in the same time duration in the previous cycle. Initial covariances of the first time slot can be estimated through distances. The longer the distance between two grids is, the larger the covariance is. As for means, before the first time slot, we can recruit more mobile users to measure environmental values in all grids and use them as initial means. This is the necessary cost of starting up the environmental monitoring project. The ground truth of each grid is set according to the real air pollution data in Beijing [2].

5.2 Performance Benchmark

For the online algorithm for the long-term problem, i.e., Alg. 1, the group of benchmarks consists of three algorithms: 1) *Cost First*: in each slot, we select mobile users in an offline way: choosing mobile users with lowest costs under the budget B_{avg}. 2) *Shortsighted UPR*: in each slot, the data utility is maximized subject to the upper bound, β, of budget. 4) *Shortsighted AVG*: the current data utility is maximized under the constraint that the total cost of selected mobile users is below B_{avg}. This strategy guarantees the average budget constraint by limiting the total cost to B_{avg}.

What's more, we compare the online algorithm for the one-slot problem, i.e., Alg. 2, with two baselines: 1) OPT: The offline optimal solution obtained by existing optimization techniques. Because the objective in the one-slot problem is submodular, it is also convex. There are many existing techniques for convex optimization. 2) Bateni: The online algorithm of Bateni et al. which computes the threshold of marginal efficiency at $\mathcal{T}/2$ by existing optimization techniques [6].

5.3 Experimental Results

5.3.1 The online algorithm for the one-slot problem. In Fig. 4 we compare our online algorithm for the one-slot problem with the optimal solution (OPT) and the algorithm of Bateni et al. (Bateni). Because OPT and Bateni need to optimize the NP-hard one-slot problem, their time complexities are very high. Thus, we set the number of mobile users to relatively small values, i.e., from 100 to 300. We just consider one slot and maximize the data utility in the slot. The upper bound of the budget is \$70.

We can see that the data utility increases with the number of mobile users because more mobile users can contribute data. The data utility achieved by our algorithm is about 0.7 times OPT and 0.9 times Bateni. Considering the time complexity, the sacrifice of data utility can be acceptable.

5.3.2 The online algorithm for the long-term problem. In Fig. 5 and Fig. 6, we report the time-averaged data utility and the time-averaged error when the number of mobile users increases from 2000 to 3000, respectively. In Fig. 5, the data utility increases with the number of mobile users. When the number of mobile users is small, more users can provide more data service. When available services are enough, the data utility can still increase because there are more mobile users with lower costs. In Fig. 6, we calculate the error as RMSE which is introduced in Section II. Errors decrease with more mobile users because there are more measurements and we can make confident estimations. Because the data utility includes both of the importance of observed grids and the error of estimation, we can see that the trend of the error is not very smooth. Shortsighted UPR has a larger data utility and a smaller error than our algorithm because it relaxes the average budget constraint. Shortsighted AVG has a lower data utility and a larger error than our algorithm because it tries to satisfy the time-averaged budget constraint by limiting the total cost in each slot to B_{avg}.

In Fig. 7 and Fig. 8, we investigate the time-averaged data utility and the time-averaged error when the average budget increases from 350 to 550. The data utility increases and the error decreases because the constraint of the average budget is relaxed. More mobile users can be recruited and more measurements are collected. We can see that the average budget has no effect on Shortsighted UPR because it does not consider this constraint.

In Fig. 9 and Fig. 10, we study the time-averaged data utility and the time-averaged cost when the upper bound β of the budget in a slot increases from \$625 to \$775. The value of β only has an effect on Shortsighted UPR (UPR) and our algorithm. The data utility increases because the larger upper bound leads to more recruited mobile users. The cost increases due to the relaxed budget constraint. The data utilities and the costs under our algorithm increase slower when β is large because the time-averaged budget cannot exceed B_{avg}.

Combining these two figures, we analyze our algorithm and the three baselines. The average data utility of Shortsighted UPR is higher than our algorithm. However, its cost is also higher. The average cost of Shortsighted UPR even exceeds B_{avg}.

In Fig. 11, we investigate how the parameter V controls the tradeoff between the data utility and the cost. With the increasing V, the time-averaged data utility is more emphasized. Thus the time-averaged utility increases and the cost becomes larger.

6 RELATED WORK

6.1 Environmental Monitoring

Environmental monitoring has been attracting much attention due to its importance in guidance on people's daily life and urban construction [18][11][29][28][23][14][15]. For example, Zheng et al. [29] aim to recover the noise map of New York city according to complaints about noises. These complaints are collected by a platform which allows people to complain about city's issues by using a mobile app or making a phone call. According to the complaint data together with social media, road network data, and Points of Interests (POIs), they make use of a context-aware tensor decomposition approach to recover the noise situation throughout New York city. However, they do not consider how to collect important data and only focus on recovering the monitoring map after all data have been collected.

He et al. [11] propose an incentive mechanism for signal map construction. By Bayesian compressive crowdsensing, they iteratively determine the selected spatial grids and predict the remaining unexplored grids. A probabilistic user participation and measurement model is applied for incentive design. However, they select the next grid to explore after the observed value of the current selected grid has been collected. What's more, they do not consider the long-term performance of the system.

Liu et al. [18] use Gaussian process to model sensing data and propose a random adaptive greedy user selection algorithm to select a set of optimal mobile users. They consider a simplified setting where the cost of each mobile user is the same. Then the knapsack constraint is changed to the cardinality constraint which makes the problem be simple. There are existing greedy algorithms for this problem [9]. What's more, they do not consider the long-term performance.

There are some other related works. Some of them [28][23] use wireless sensor networks to collect data. They usually study where to place sensors before network deployment. After the network is deployed, the locations of sensors cannot be changed. Some of them [14][15] only collect data and do not estimate missing data. Therefore, they cannot be applied to our problem.

6.2 Data Collection by Crowdsourcing

There are many works about data collection by crowdsensing [24][17][21][25][10][22]. For example, Han et al. [10] study the total revenue maximization problem under the budget constraint in a multi-round scenario. The decision is which worker should perform which task. In a round, the decision is made by an offline way. Each round has the same candidate set of workers. They assume that the average revenue that a worker brings by collecting one data instance is not known and needs to be learned in the process. They propose a learning algorithm to get more information about the revenue of each worker and try to minimize the regret (the difference between the total revenue achieved by their algorithm and the optimal solution).

In another example, Wang et al. [22] want to select a set of mobile users who have considerable expertise and whose total cost is smaller than the budget. They consider the long-term performance of the system. However, the transformed one-slot problem is not NP-hard. They can use existing optimization techniques to solve it. Because the one-slot problem is easier, the transformation from the long-term problem to a series of one-slot problems is also easier.

7 CONCLUSION

This paper focuses on selecting optimal mobile users for long-term environmental monitoring by crowdsourcing. Gaussian Process is applied to infer the environmental values of unobserved grids and construct the data utility model. The mobile user selection problem is formulated as the time-averaged data-utility maximization under the time-averaged budget constraint. This problem is particularly challenging because we have to make decisions without the future information and maximize a non-monotone submodular objective under the budget constraint in each slot. To address these challenges, we first make use of Lyapunov optimization to decompose the long-term optimization problem into a series of real-time optimization problems which do not require a priori knowledge about the future information. We then propose a time-efficient online algorithm to solve the NP-hard utility-minus-drift problem in each slot.

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A PROOF OF THEOREM 3.1

Because of the fact that $(max[\alpha, 0])^2 \leq \alpha^2$ for any α, for the Lyapunov drift, we have

$$\begin{aligned}\Delta(Q(t)) &= \frac{1}{2}[Q^2(t+1) - Q^2(t)] \\ &\leq \frac{1}{2}\{[Q(t) + C_t - B_{avg}]^2 - Q^2(t)\} \\ &\leq \frac{1}{2}[C^2(t) + B_{avg}^2] + Q(t)[C_t - B_{avg}].\end{aligned} \tag{16}$$

Combining the data utility in slot t and taking an expectation, we can get the lower bound of the utility-minus-drift function as follows.

$$\begin{aligned} &VG(S_t) - \Delta(Q(t)) \geq \\ &VG(S_t) - Q(t)\mathbb{E}\{C_t - B_{avg}|Q(t)\} - D, \end{aligned}$$

where $D = \frac{1}{2}(\beta^2 + B_{avg}^2)$ is a constant value. Hence, the theorem is proved.

B PROOF OF THEOREM 3.3

Assume that mobile user sets S_t^1 and S_t^2 belong to U^t, $S_t^1 \subseteq S_t^2$ and $u_i^t \in U^t \backslash S_t^2$. We prove that $G(S_t^1 \cup \{u_i^t\}) - G(S_t^1) \geq G(S_t^2 \cup \{u_i^t\}) - G(S_t^2)$. The value of $G(S_t^1 \cup \{u_i^t\}) - G(S_t^1)$ is

$$\begin{aligned} &Ve_{\hat{l}_i^t} - Q(t)c_i^t + VWH(X_{R_t^1 \backslash \hat{l}_i^t}) - VWH(X_{R_t^1 \backslash \hat{l}_i^t}|X_{A_t^1 \cup \hat{l}_i^t}) \\ &- [VWH(X_{R_t^1}) - VWH(X_{R_t^1}|X_{A_t^1})]. \end{aligned} \tag{17}$$

The value of $G(S_t^2 \cup \{u_i^t\}) - G(S_t^2)$ is

$$\begin{aligned} &Ve_{\hat{l}_i^t} - Q(t)c_i^t + VWH(X_{R_t^2 \backslash \hat{l}_i^t}) - VWH(X_{R_t^2 \backslash \hat{l}_i^t}|X_{A_t^2 \cup \hat{l}_i^t}) \\ &- [VWH(X_{R_t^2}) - VWH(X_{R_t^2}|X_{A_t^2})]. \end{aligned} \tag{18}$$

Then, we have

$$\begin{aligned} &G(S_t^1 \cup \{u_i^t\}) - G(S_t^1) - [G(S_t^2 \cup \{u_i^t\}) - G(S_t^2)] \\ =&VW[H(X_{R_t^1 \backslash \hat{l}_i^t}) - H(X_{R_t^2 \backslash \hat{l}_i^t})] \\ &- VW[H(X_{R_t^1 \backslash \hat{l}_i^t}|X_{A_t^1 \cup \hat{l}_i^t}) - H(X_{R_t^2 \backslash \hat{l}_i^t}|X_{A_t^2 \cup \hat{l}_i^t})] \\ &- VW[H(X_{R_t^1}) - H(X_{R_t^2})] \\ &+ VW[H(X_{R_t^1}|X_{A_t^1}) - H(X_{R_t^2}|X_{A_t^2})]. \end{aligned}$$

The chain-rule of entropies shows that for any random variables X, Y, Z, $H(X,Y) = H(X|Y) + H(Y)$ and $H(X,Y|Z) = H(X|Y,Z) + H(Y|Z)$. According to the relationship among S_t^1, S_t^2, u_i^t, and U^t, we can get the relationship among L, R_t^1, R_t^2, A_t^1, A_t^2 and $\hat{l}_i^t$ which is shown in Fig. 12. Then, we have $H(X_{R_t^1 \backslash \hat{l}_i^t}) = H(X_{R_t^1 \backslash R_t^2}|X_{R_t^2 \backslash \hat{l}_i^t}) + H(X_{R_t^2 \backslash \hat{l}_i^t})$ because $R_t^1 \backslash \hat{l}_i^t = \{R_t^1 \backslash R_t^2\} \cup \{R_t^2 \backslash \hat{l}_i^t\}$ and $X_{R_t^1 \backslash \hat{l}_i^t} = (X_{R_t^1 \backslash R_t^2}, X_{R_t^2 \backslash \hat{l}_i^t})$. Combining the chain-rule of entropies and Fig. 12, we can get results of the other three subtraction formulas in the above equation and conclude:

$$\begin{aligned} &G(S_t^1 \cup \{u_i^t\}) - G(S_t^1) - [G(S_t^2 \cup \{u_i^t\}) - G(S_t^2)] \\ =&VWH(X_{R_t^1 \backslash R_t^2}|X_{R_t^2 \backslash \hat{l}_i^t}) - VWH(X_{A_t^2 \backslash A_t^1}|X_{A_t^1 \cup \hat{l}_i^t}) \\ &- VWH(X_{R_t^1 \backslash R_t^2}|X_{R_t^2}) + VWH(X_{A_t^2 \backslash A_t^1}|X_{A_t^1})] \geq 0. \end{aligned} \tag{19}$$

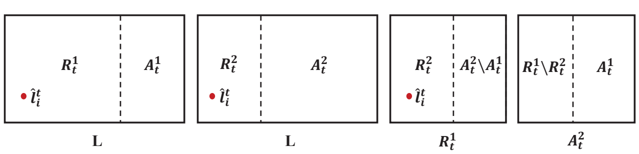

Figure 12: Environmental monitoring by crowdsourcing.

C PROOF OF THEOREM 4.1

There exists an optimal mobile user selection policy achieving the optimal time-averaged data utility F^{opt} [19]. Under this policy, the data utility in each slot is F_t^{opt} and the total cost in each slot is C_t^{opt}. The average total cost satisfies

$$\mathbb{E}\{C_t^{opt}\} \leq B_{avg} - \gamma. \tag{20}$$

In each slot t, with the optimal solution of the one-slot problem, the data utility is F_t^* and the total cost is C_t^*. Then,

$$\begin{aligned} &V\mathbb{E}\{F(S_t)|Q(t)\} - \Delta(Q(t)) \\ &\geq V\mathbb{E}\{F(S_t)|Q(t)\} - Q(t)\mathbb{E}\{C_t - B_{avg}|Q(t)\} - D \\ &\geq eV\mathbb{E}\{F_t^*|Q(t)\} - eQ(t)\mathbb{E}\{C_t^* - B_{avg}|Q(t)\} - D \\ &\geq eV\mathbb{E}\{F_t^{opt}|Q(t)\} - eQ(t)\mathbb{E}\{C_t^{opt} - B_{avg}|Q(t)\} - D. \end{aligned} \tag{21}$$

The first two inequalities are due to Theorem 3.1 and the approximation ratio of Alg. 2. The third inequality is derived from the fact that the optimal solution of the one-slot problem achieves the largest lower bound. Then,

$$\begin{aligned} &V\mathbb{E}\{F(S_t)|Q(t)\} - \Delta(Q(t)) \\ &\geq eVG^{opt} + e\gamma Q(t) - D. \end{aligned} \tag{22}$$

Taking an expectation with respect to the distribution of $Q(t)$, and then we can obtain the following inequality by the iterative expectation law.

$$\begin{aligned} &V\mathbb{E}\{F(S_t)\} - \mathbb{E}\{L(Q(t+1))\} + \mathbb{E}\{L(Q(t))\} \\ &\geq eVG^{opt} + e\gamma\mathbb{E}\{Q(t)\} - D. \end{aligned} \tag{23}$$

Adding up the above inequality over all slots and dividing each side by the total time T, we can get

$$\begin{aligned} &V\tfrac{1}{T}\textstyle\sum_{t=0}^{T-1}\mathbb{E}\{F(S_t)\} - \frac{\mathbb{E}\{L(Q(T))\} - \mathbb{E}\{L(Q(0))\}}{T} \\ &\geq eVG^{opt} + \tfrac{e\gamma}{T}\textstyle\sum_{t=0}^{T-1}\mathbb{E}\{Q(t)\} - D. \end{aligned} \tag{24}$$

By deleting the non-negative terms $L(Q(T))$ and $Q(t)$,

$$V\tfrac{1}{T}\textstyle\sum_{t=0}^{T-1}\mathbb{E}\{F(S_t)\} + \frac{\mathbb{E}\{L(Q(0))\}}{T} \geq eVG^{opt} - D.$$

When $T \to \infty$, the theorem is proved.

$$\lim_{T\to\infty} \tfrac{1}{T} \cdot \textstyle\sum_{t=0}^{T-1}\mathbb{E}\{F(S_t)\} \geq eG^{opt} - D/V. \tag{25}$$

D PROOF OF THEOREM 4.2

We have $\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{F(S_t)\} = F \leq F^{opt}$. What's more, $L(Q(T))$ and each $Q(t)$ are non-negative. According to inequality (24), we get

$$VG^{opt} + \frac{\mathbb{E}\{L(Q(0))\}}{T} \geq eVG^{opt} + \tfrac{e\gamma}{T}\textstyle\sum_{t=0}^{T-1}\mathbb{E}\{Q(t)\} - D.$$

By rearranging the inequality, we have

$$\tfrac{1}{T}\textstyle\sum_{t=0}^{T-1}\mathbb{E}\{Q(t)\} \leq \frac{(1-e)VG^{opt}+D}{e\gamma} + \frac{\mathbb{E}\{L(Q(0))\}}{e\gamma T}.$$

When $T \to \infty$, the theorem is proved.

$$\lim_{T\to\infty} \tfrac{1}{T}\textstyle\sum_{t=0}^{T-1}\mathbb{E}\{Q(t)\} \leq [(1-e)VG^{opt} + D]/(e\gamma).$$