Bitcoin Mining with Transaction Fees
A Game on the Block Size

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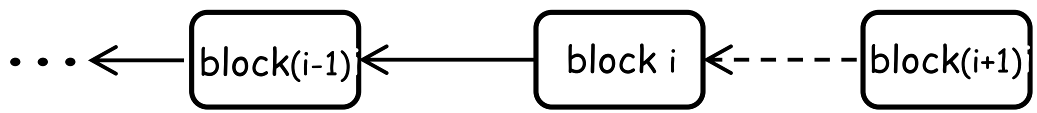
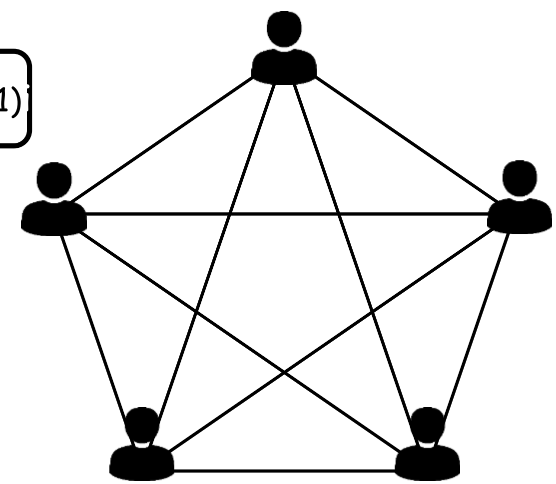
1. Bitcoin

- A blockchain-based digital payment system
  - A distributed ledger using PoW mining mechanism
    - Prob. of solving a block puzzle relies on a miner's computing rate

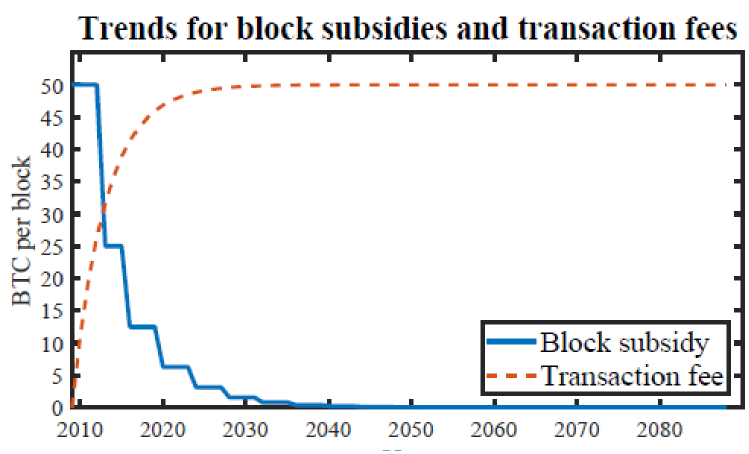
      λ_i= individual power / total power
  - To win a block
    - Solve puzzle and then propagate the block to reach consensus
    - Propagation delay discounts the winning probability W_i





Bitcoin Mining Incentives

- Each winner will be rewarded with R_i, including
  - Block subsidies S: finite supply and eventually become zero
  - Transaction fees F_i: offered by users and gradually increase
    - Without F_i, miners have no incentive to include transactions in their blocks [1]
- Trend between S and F_i
  - The sum of block subsidies and the average transaction fees collected per block remains constant [2].



Block reward evolution trend[2].

[1] Houy, Nicolas. "The Bitcoin mining game." SSRN Electronic Journal, 2014.
[2] Kaskaloglu, Kerem. "Near zero Bitcoin transaction fees cannot last forever." Proceedings of the International Conference on Digital Security and Forensics, 2014

Miner's Utility U_i

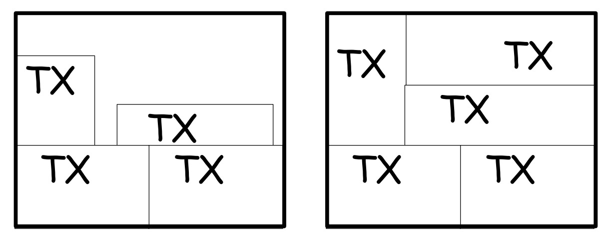
- Utility $U_i = R_i \times W_i$
  - Block reward $R_i = S + F_i$
    - Block subsidy S is a fixed value in a block
    - TX fee density
    - Transaction (TX) fee $F_i \propto$ block size: $F_i = \alpha B_i$
  - Winning probability W_i
    - Positively related to computing rate λ_i
    - Discounted by propagation time p_i

Network delay rate

where $p_i \propto$ block size: $p_i = \beta B_i$ [3]

- Block size B_i
  - Default size $\bar{B} = 1$ MB
    - Recommended by system
    - Miner can choose any $B_i \leq \bar{B}$

1 MB in total | 1 MB in total



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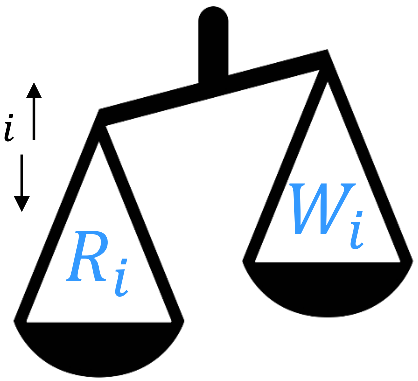
[3] Decker, Christian, and Roger Wattenhofer. "Information propagation in the bitcoin network." *IEEE P2P 2013 Proceedings*. IEEE, 2013.

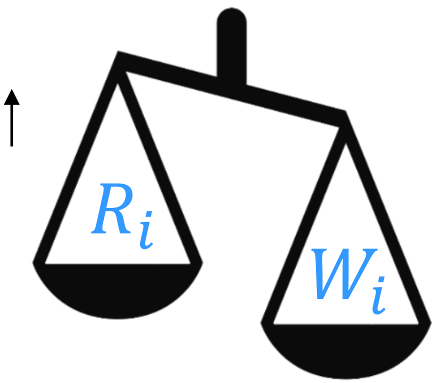
Trade-off on Block Size

- Choose a large block size\ a small block size

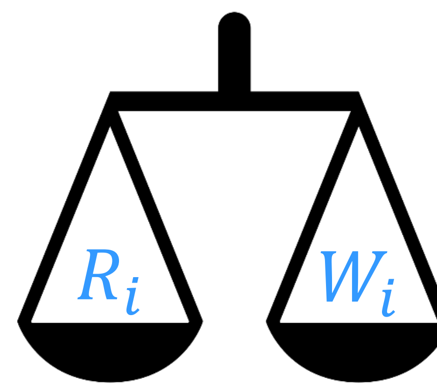
If $B_i \uparrow$
then $R_i \uparrow$
but $W_i \downarrow$



If $B_i \downarrow$
then $W_i \uparrow$
but $R_i \downarrow$



- Find an optimal size B_i to maximize U_i
  - We want to find a suitable $\bar{B}$ such that
    - $\bar{B}$ is each miner's optimal size



2. Characterize W_i Using B_i

- Distribution of block finding time X_i
  - PDF: $f_{X_i}(t; B_i, \lambda_i) = \begin{cases} 0 & t < p_i \\ \lambda_i e^{-\lambda_i(t-p_i)} & t \geq p_i \end{cases}$
  - CDF: $F_{X_i}(t; B_i, \lambda_i) = \begin{cases} 0 & t < p_i \\ 1 - e^{-\lambda_i(t-p_i)} & t \geq p_i \end{cases}$
- W_i among n miners
  - Winner should have the smallest block finding time
  - $$W_i = Pr\left(X_i = min\left\{X_j | j = 1, \cdots n\right\}\right)$$
    $$= \lambda_i \boxed{\sum_{l=i}^{n} \frac{e^{\sum \lambda_j (p_j - p_l)} - e^{\sum \lambda_j (p_j - p_{l+1})}}{\sum \lambda_j}}$$

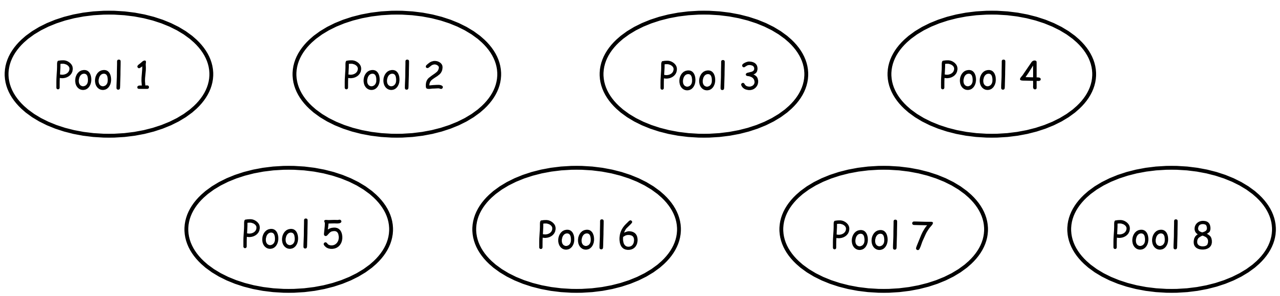
Discounted by propagation delay

3. Game on Block Size

- Two types of players
  - Cheater: manipulate his block size B_i for utility maximization
  - Honest miner: use default block size $\bar{B}$
- Game analysis on two different settings
  - Homogeneous miners
    - Assume all miners have the same computing rate
    - Analysis on Bitcoin mining network
  - Heterogeneous miners
    - Each miner can have different computing rate
    - Case studies on one cheater and two cheaters

4. Homogeneous Setting

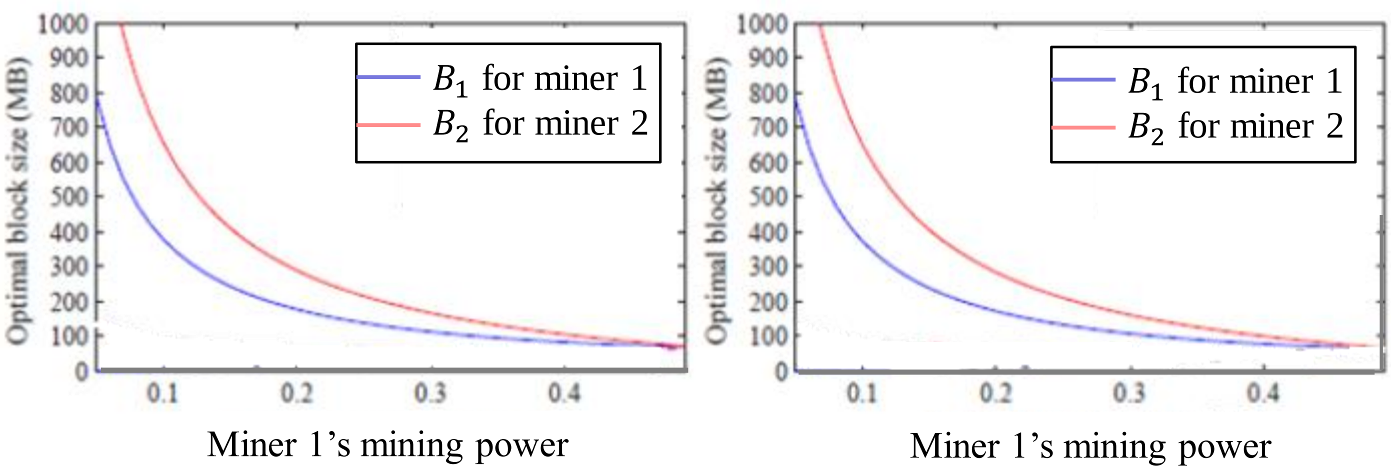
- Bitcoin mining network
  - Approximated as 8 equal-size pools [4]
    - Viewed as 8 homogeneous cheaters
  - $S = 12.5$ and $F_i = B_i$ (that is $\alpha = 1$)
  - Theorem 1. In an 8-pool Bitcoin mining network, all cheaters' optimal block size is 4MB.
    - Thus, we recommend 4MB as default block size



[4] Tsabary, Itay, and Ittay Eyal. "The gap game." Proceedings of the 2018 ACM SIGSAC Conference on Computer and Communications Security. ACM, 2018.

5. Heterogeneous Setting

- Qualitative analysis on utility and block size
  - Theorem 2. A miner indirectly increases each of his rivals' utility by increasing his own block size.
  - Theorem 3. A miner's optimal block size is positively related to his computing power (Fig. 1)

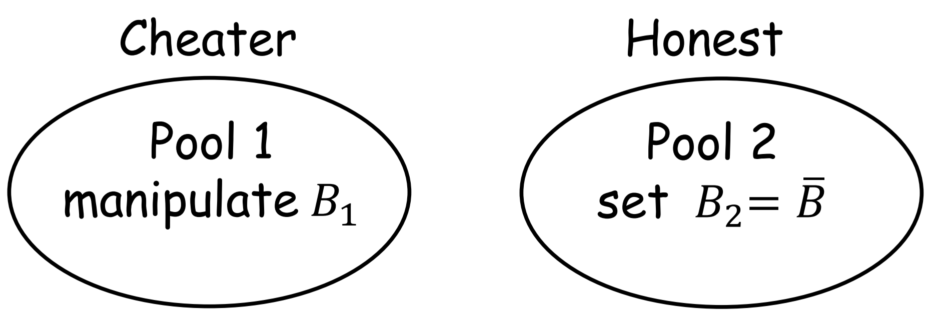


(a) $S = 12.5, \alpha = 0.16, \beta = 8.2$ (b) $S = 25, \alpha = 0.3, \beta = 8.2$

Fig. 1: Two miners 1 & 2: $\lambda_1 < \lambda_2$, $\lambda_1 + \lambda_2 = 1$

Case Study: One Cheater

- Setting: miners are divided into two groups
  - Corrupted pool controlled by a cheater: Pool 1
    - Optimize B_1 for utility maximization
    - Computing rate: λ_1
  - The rest of the miners are honest: Pool 2
    - Use the default block size $\bar{B}$
    - Computing rate: λ_2 in total



Pool 1 and pool 2 are heterogeneous
with regard to computing rate.

Pool 1's Utility Analysis

- Parameters affecting pool 1's optimal size
  - B_1 is positively related to computing rate λ_1
  - Decrease of subsidy S leads to increase of B_1
  - Large network delay rate β will reduce B_1

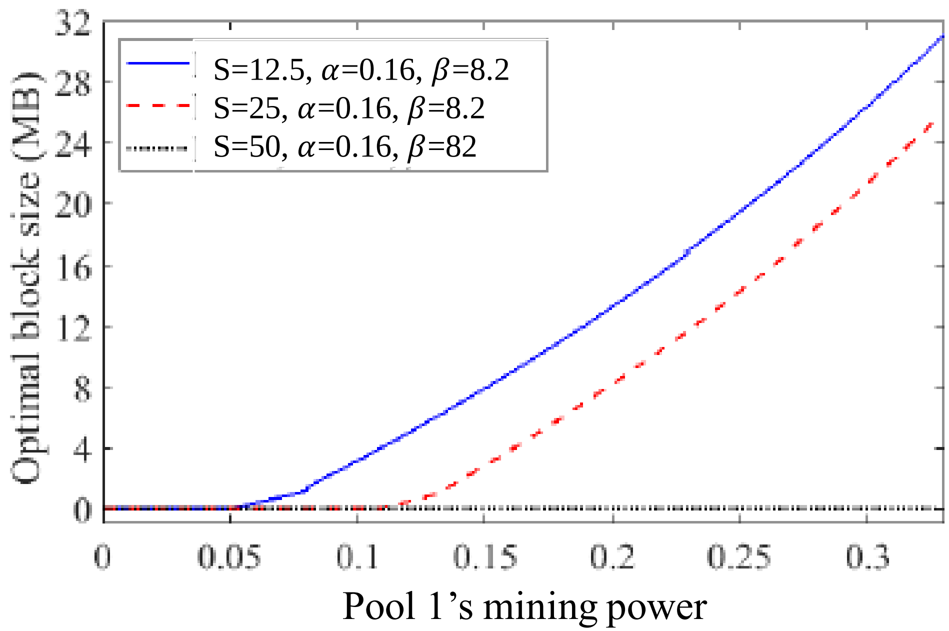
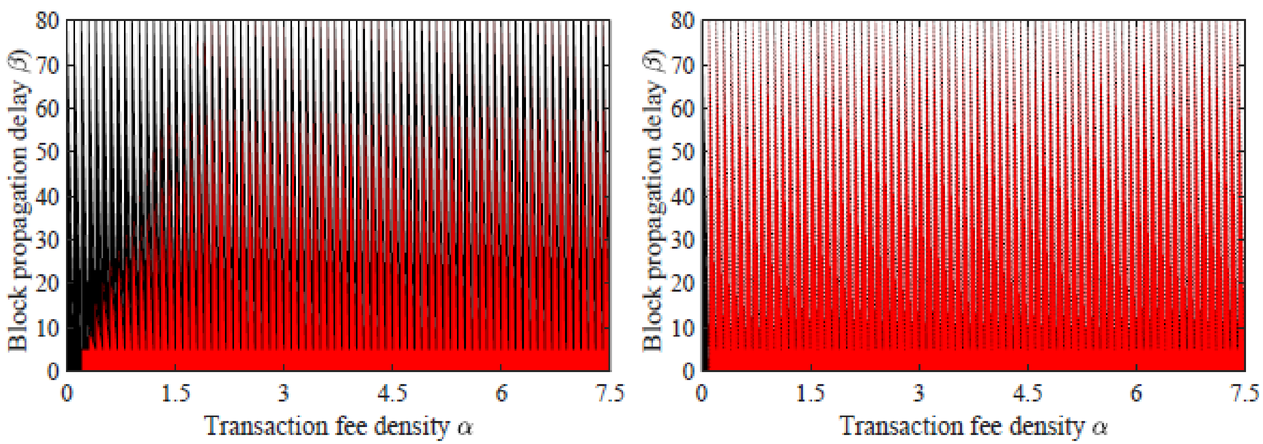


Fig. 2: Optimal block size using different sets of (S, α, β)

Peaceful Equilibrium

- Peaceful equilibrium is a condition where
  - Pool 1's optimal block size $B_1 = \bar{B}$
- Upper bound of λ_1
  - $\boxed{\text{Theorem 4.}}$ If $\lambda_1 \leq 1/3$, A's optimal block size B_1 equals to $\bar{B}$
- Block subsidy and equilibrium ($\lambda_1 > 1/3$)
  - The decrease of S could lead to more equilibria (Fig. 3)
    - Since TX fees become main income, pool 1 has incentive to increase B_1



S = 12.5 S = 0

Fig. 3: Red area represents $B_1 = \bar{B}$ and black area represents $B_1 < \bar{B}$

Network Delay and Equilibrium ($\lambda_1 > 1/3$)

- When network delay is reasonable: (Fig. 4)
  - If α is high enough and S is low, then $B_1 = \bar{B}$
- When network delay is serious: (Fig. 5)
  - Hard to see peaceful equilibrium, that is $B_1 < \bar{B}$
  - Damage Bitcoin network if attackers issue delay attacks

Fig. 4: $\beta = 8.2$

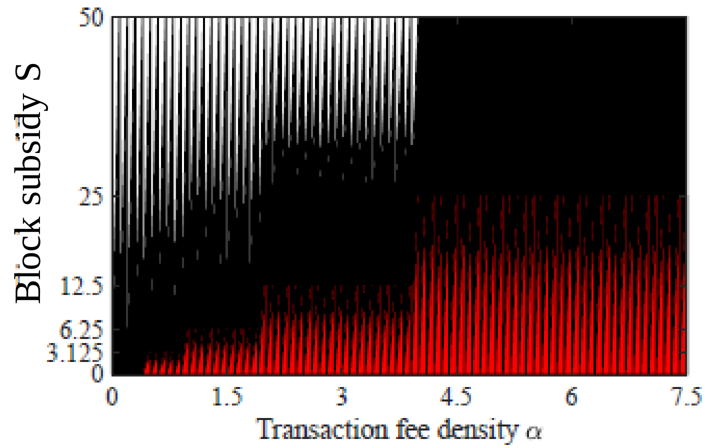


Fig. 5: $\beta = 82$

Case Study: Two Cheaters

- Setting: miners are divided into three groups
  - Two cheaters: L and H
    - L has a smaller pool with computing rate: λ_L
    - H has a larger pool with computing rate: λ_H
  - The rest of the miners M are honest
    - Use the default block size $\bar{B}$ with computing rate: λ_M in total

Cheater	Cheater	Honest
Pool L manipulate B_L	Pool H manipulate B_H	Pool M set $B_M = \bar{B}$

L, H, and M are heterogeneous regarding to computing rate.

Sided Misbehaviors

- One side: only L cheats on his block size
  - If $\lambda_L > 8\%$, L's optimal size $B_L < \bar{B}$ (Fig. 6)
- Both sides: L and H cheat on block sizes
  - For $\bar{B} = 1$ MB, L and H always have optimal sizes smaller than $\bar{B}$, no matter what their computing rates are (Fig. 7)
  - Current default size must be redefined

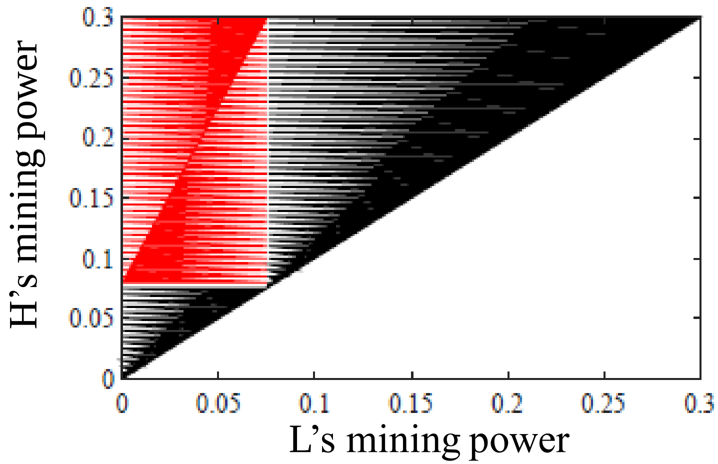


Fig. 6

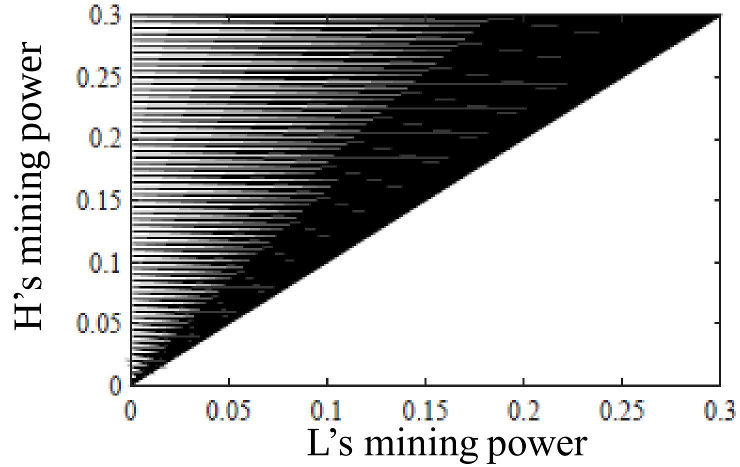


Fig. 7

6. Conclusion

- A game on block size
  - Consider tradeoff between propagation time and TX fees
  - Model the relation between winning probability and block size
- Game Analysis on two different settings
  - Homogeneous miners in bitcoin mining network
  - Heterogeneous miners for case studies
- Real-world data to confirm theoretical analysis
  - Future work: conduct experiments on real blockchain platform, eg. CITA [5], to measure real-time propagation delay influences.

[5] "CITA." [Online]. Available: https://github.com/cryptape/cita

Thank you

Q & A