Hierarchical Edge-Cloud Computing for Mobile Blockchain Mining Game

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***Abstract*—Computation offloading has been considered as a viable solution to blockchain mining in mobile environments. In this paper, we present a two-layer computation offloading paradigm that includes an edge computing service provider (ESP) and a cloud computing service provider (CSP). We formulate a multi-leader multi-follower Stackelberg game to address the computing resource management problem in such a network, by jointly maximizing the profits of each service provider (SP) and the payoffs of individual miners. Two practical scenarios are investigated: a fixed-miner-number scenario for permissioned blockchains and a dynamic-miner-number scenario for permissionless blockchains. For the fixed-miner-number scenario, we discuss two different edge operation modes, *i.e.*, the ESP is *connected* (to the CSP) or *standalone*, which form different miner subgames based on whether each miner's strategy set is mutually dependent. The existence and uniqueness of Stackelberg equilibrium (SE) in both modes are analyzed, according to which algorithms are proposed to achieve the corresponding SE(s). For the dynamic-miner-number scenario, we focus on the impact of population uncertainty and find that the uncertainty inflates the aggressiveness in the ESP resource purchasing. Numerical evaluations are presented to verify the proposed models.**

***Index Terms*—Cloud computing, edge computing, game theory, load sharing, mobile blockchain mining.**

I. Introduction

There is a wide adoption of blockchain technology ranging from cryptocurrency, financial services, Internet of Things (IoT) to public and social services. As a distributed ledger, blockchain records data in the form of linked blocks secured by cryptography. Consensus protocol is the core of blockchain, since it regulates the maintenance for such an append-only public ledger in a distributed fashion. Currently, most blockchain applications are on top of a proof-of-work (PoW) protocol. In a PoW-based blockchain network, miners collect blocks of data, verify their integrity, and append them to the blockchain. In order to add a block to the blockchain, miners are required to solve a computationally challenging PoW puzzle. The security and reliability are thus ensured by this mechanism which requires numerous trials and errors for a valid solution. The blockchain grows with the repetitive block-appending processes, each of which is considered as one mining round; meanwhile, the owner of the on-chain block receives monetary rewards as the mining incentive.

However, the energy consumption and the computing power required to perform PoW computation are prohibitively high for mobile devices, thus hindering the practical usage of blockchain in mobile environments. Offloading PoW computation to the external machines has been proven effective in overcoming the aforementioned limitations and promoting mobile blockchain applications. Specifically, both an *edge computing service provider* (ESP) and a *cloud computing service provider* (CSP) can provide computing resources for mobile devices. While a CSP can guarantee a good isolation among multiple computation offloading requests (*i.e.*, there is no competition for cloud computing resources) with a relatively cheap price, significant network delays hamper the performance of cloud computing. Due to the delay-sensitive nature of mining, an ESP is considered as an efficient proximity alternative with the capability of providing low-latency service. However, mobile miners may have to compete against each other for the limited and expensive edge computing resources.

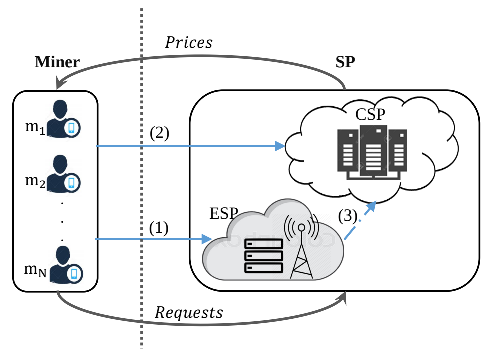

Fig. 1: Mobile blockchain mining network: (1) offloading to the ESP; (2) offloading to the CSP; (3) automatic transfer from the ESP to the CSP.

In this paper, we present a hierarchical computation offloading paradigm consisting of two service providers (SPs), *i.e.*, a nearby ESP and a remote CSP, and a set of miners in a mobile blockchain mining network. As depicted in Fig. 1, each miner is willing to offload its PoW computation to either of these two SPs or both of them. Once the ESP is overloaded with requests, it responds differently according to its operation mode. Specifically, two edge computing operation modes, *i.e.*, the ESP *connected* to the CSP and *standalone*, have been implemented in practice. Consequently, an edge computing request, which fails to be satisfied by the ESP, will be sent to the backup CSP in the connected mode (characterized by the line(3) in Fig. 1), or completely rejected in the standalone mode. Miners' requests are mutually affected in the standalone mode, and should be dedicated to avoid overloading the ESP.

We exploit game theory to analyze the complex interactions

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among SPs and mobile miners. To solve the price-based resource management problem, we leverage a multi-leader multi-follower Stackelberg game, which includes two subgames for the SPs (as leaders) and the miners (as followers), respectively. In the SP subgame, each SP has a privilege to set unit prices on its computing resources by anticipating the miners' responses. In the miner subgame, the miners decide their requests according to the observed unit prices. Moreover, we investigate how edge operation modes will affect the miner subgame. In the connected mode, the miner subgame is formulated as a classical Nash equilibrium problem (NEP). However, the miner subgame becomes a generalized Nash equilibrium problem (GNEP) in the standalone mode. GNEPs differ from NEPs in that, while in an NEP only the players' objective functions depend on the other players' strategies, in a GNEP both the objective functions and the strategy sets depend on the other players' strategies. In the standalone mode, due to the limited computing units at the ESP side, whether a miner's edge computing request can be satisfied is affected by other miners' requests.

All previous studies assume that the miner number is fixed as a common knowledge in the proposed games. In practice, for permissionless blockchains where miners can randomly join or leave, the miner number may change. Thus, we also discuss the impact of population uncertainty on the miners' strategies by modeling the miner number as a random variable. The major contributions of this paper are as follows:

- We propose a Stackelberg game to solve a price-based computing resource management problem in a mobile blockchain mining network with two SPs.
- We study the proposed Stackelberg game in two practical edge operation modes, thereby formulating two different miner subgames: an NEP in the connected mode and a GNEP in the standalone mode.
- We analyze the existence and uniqueness of Stackelberg equilibrium (SE) for both edge operation modes, based on which algorithms are proposed to obtain SE solutions.
- We consider a special case of homogeneous miners and derive explicit-form expressions of the most profitable price strategies for each SP and the optimal resource requests for individual miners in each mode.
- We study the impacts of population uncertainty, which incurs more resource requests at the ESP side.
- We conduct experiments in a reinforcement learning framework to validate our analysis. The achieved equilibria are consistent with our theoretical results.

II. System Model and Game Formulation

A. A Mobile Blockchain Mining Network

This paper focuses on a mobile blockchain mining network. Corresponding notations are listed in Table I. We consider N end users, which we also call miners, and two service providers. Fig. 1 depicts an overview of this network. The SP side consists of a nearby ESP and a remote CSP that make profits by contributing their computing power, sold by unit. One unit from the ESP is functionally equivalent to one from the CSP. In the proposed network, message transmission time is viewed as communication delay. We neglect the communication delay between the ESP and miners and define communication delay between the CSP and the ESP/miners is D_c. Besides, the ESP is assumed to have limited computing capability, while the CSP owns unlimited computing power.

TABLE I: Summary of Notations.

Symbol	Description
P_e/P_c	unit price set by the ESP/the CSP
C_e/C_c	unit cost of the ESP/the CSP
V_e/V_c	utility of the ESP/the CSP
$h/1-h$	the ESP's expected hit/miss rate in the connected mode
E_{max}	total computing capacity of the standalone ESP
D_c	average delay the CSP
N	total number of miners
m_i	the i-th miner
$U_i/W_i/B_i$	m_i's utility/winning probability/budget
e_i/c_i	number of ESP/CSP units requested by m_i
r_i	m_i's request vector to the SPs, in the form of $[e_i, c_i]^\intercal$
$\boldsymbol{r}$	stacked request vectors of all miners
$\boldsymbol{r}_{-i}$	stacked request vectors of all miners excluding m_i's
R	blockchain mining reward
β	discount rate caused by delay

The end-user side is a network with N miners using mobile devices. We differentiate them in terms of available budget which gives an upper bound on the amount of computing units they can afford. Thus, different types of miners have different requests on computing power. We employ a utility function to describe each miner's expected payoff, *i.e.*, the difference between its expected income and expected cost. The SPs and the miners have bidirectional communications for exchanging price and request information. Miners receive prices from the SPs and transmit their requests to them.

We consider two practical edge operation modes, *i.e.*, connected to the CSP or standalone, differing in whether the ESP would share loads with the CSP if it is computationally overloaded. Based on these two modes, we characterize the limited computing capability of the ESP in two ways. In the connected mode, to capture the capacity limitation, we define a hit (miss) rate h $(1-h)$ to represent the probability of an edge request being satisfied by the ESP (being transferred to the CSP), where h is a common knowledge in the game. In the standalone mode without load sharing, the ESP only has E_{max} computing units and hence rejects requests once overloaded.

B. SP-Miner Interaction: A Stackelberg Game

We focus on interactions between the SPs and the miners. Each miner's income depends all miners' strategies and its cost varies according to the prices set by each SP. In fact, each SP decides its unit price by considering miners' requests as well as the rival SP's price. Game theory provides a natural paradigm to model the interactions between the SPs and the miners in this network. Each SP sets the unit price and announces it to the miners. The miners respond to the price by requesting an optimal amount of computing units to the SPs. Since the SPs act first and then the miners make their decision based on the prices, the two events are sequential. Thus, we model the interactions between the SPs and the miners using a Stackelberg game. In our proposed game, the SPs are the leaders and the consumers are the followers. It is a multi-leader multi-follower Stackelberg game with two stages.

In the first stage, the competition between the ESP and the CSP forms a non-cooperative leader subgame, where each SP optimizes its unit price (P_e/P_c) by predicting the miners' reactions as well as considering the other SP's price strategy. In the second stage, each miner, *i.e.*, m_i, responds to the current prices, by sending request(s) to the target SP(s), considering its budget B_i and requests of other miners'. Since requests are generated for individual utility maximization, a non-cooperative follower subgame is also formed.

1) Miner Side Utility: Let e_i and c_i be m_i's requests on the ESP and the CSP, respectively. Given the constant R as the mining reward, we define m_i's optimization problem below.

Problem 1 (MINER SUBGAME: $\mathrm{OP}_{\mathrm{MINER}}$).

$$\textit{maximize} \quad U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i), \tag{1a}$$

$$\textit{subject to} \quad P_e \cdot e_i + P_c \cdot c_i \leq B_i, \quad e_i \geq 0, \quad c_i \geq 0, \tag{1b}$$

where W_i represents m_i's expected winning probability, an accurate definition and detailed explanations of which will be given in Section III. Each miner m_i aims to maximize its utility and constraint (1b) ensures that m_i is within its budget.

2) SP Side Utility: The objective of each SP is to optimize its profit by determining the corresponding unit price. Given the ESP's unit cost C_e and the CSP's unit cost C_c, the optimization problem (including $\mathrm{OP}_{\mathrm{ESP}}$ and $\mathrm{OP}_{\mathrm{CSP}}$) at SP side is thus defined as in Eq.(2a) and Eq.(2b) for the ESP and the CSP, respectively.

Problem 2 (SP SUBGAME: $\mathrm{OP}_{\mathrm{SP}}$).

$$\textit{maximize} \quad V_e = (P_e - C_e) \cdot E \quad \textit{where } E = \sum_{i=1}^{N} e_i \tag{2a}$$

$$\textit{maximize} \quad V_c = (P_c - C_c) \cdot C \quad \textit{where } C = \sum_{i=1}^{N} c_i \tag{2b}$$

3) Stackelberg Game: $\mathrm{OP}_{\mathrm{SP}}$ and $\mathrm{OP}_{\mathrm{MINER}}$ together form the proposed Stackelberg game. To achieve the corresponding Stackelberg equilibrium (SE) in this game, where neither the leaders (SPs) nor the followers (miners) have incentives to deviate, we need to find its subgame perfect Nash equilibria (NE) in both the leader stage and the follower stage, by applying backward induction. Formally, the SE point(s) is defined as follows.

Definition 1. *Let $[E^*, C^*]$ and $[P_e^*, P_c^*]$ denote the optimal resource request vector of all miners and the optimal computing unit price vector of SPs, respectively. Let $[e_i^*, c_i^*]_{i=1}^N = [E^*, C^*]$, then the point (E^*, C^*, P_e^*, P_c^*) is the Stackelberg equilibrium if the following conditions hold:*

$$V_e(P_e^*, E^*) \geq V_e(P_e, E^*), \forall P_e, \tag{3a}$$

$$V_c(P_c^*, C^*) \geq V_c(P_c, C^*), \forall P_c, \tag{3b}$$

$$U_i(e_i^*, c_i^*, P_e^*, P_c^*) \geq U_i(e_i, c_i, P_e^*, P_c^*), \forall i. \tag{3c}$$

C. Main Results

We summarize the main results of our analysis based on whether the miner number is a constant or a random variable.

Scenario 1: the miner number N is a constant.

(1) The ESP operates in the connected mode:

- In the heterogeneous-miner case, we prove the existence and uniqueness of SE (Theorem 3) and provide a best response algorithm (Algorithm 1) to find the unique SE point.
- In the homogeneous-miner case, we derive explicit-form expressions of the optimal pricing for the SPs (Theorem 5) and resource management strategies (Theorem 4) for all the miners, given miners share an identical budget.

(2) The ESP operates in the standalone mode:

- In the heterogeneous-miner case, the existence of the SE is validated by capitalizing on the variational inequality theory (Theorem 6). An effective distributed price bargaining algorithm (Algorithm 2) with guaranteed convergence is proposed to find one SE point.
- In the homogeneous-miner case, we symbolically express the optimal prices and resource management strategies (Table II) given that each miner has unlimited budget.

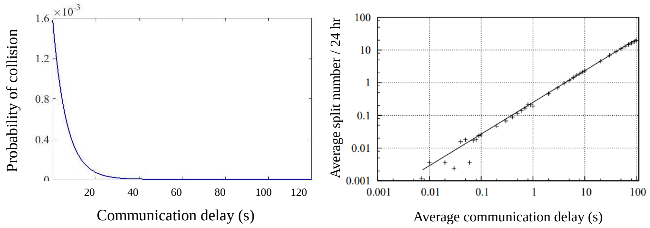
Scenario 2: the miner number N is a random variable.

- Assuming the miner number is subject to a Gaussian distribution, we reformulate the proposed game and apply a modified version of Algorithm 2 to achieve one SE point, the correctness of which is further confirmed by a reinforcement learning framework.
- Experiments indicate population uncertainty renders miners more aggressive to buy resources from the ESP.

III. A MINER'S WINNING PROBABILITY

A. Parameter Analysis

As the core part of each miner m_i's utility, W_i is determined by multiple parameters. To win mining rewards, m_i has to be the first to solve its PoW puzzle and propagate its block to reach consensus. The chance for m_i to find a PoW solution is positively correlated to its relative computing power, which is the ratio of m_i's computing power out of all computing power in the network. There is a delay for a mined block to be known by the entire network. During the delay period, another conflicting block may be found and propagates in the network as well. An earlier-mined block can be nullified since its conflicting block may reach consensus faster. Generally, delays may cause the occurrence of conflicting blocks, and then lower the probability of a mined block being accepted by the blockchain. Obviously, W_i is discounted by delays. The relation between the probability of block collision and the delay has been studied in Bitcoin [1], a classic PoW-based blockchain application. Fig. 2(a) provides its block collision probability density function (PDF) with respect to the communication delay, which is subject to an exponential distribution. Thereby, the discount rate, *i.e.*, the block collision cumulative distribution function (CDF), is almost linear to the communication delay, as shown in Fig. 2(b). In this paper, we assume that the proposed network follows the same pattern of collision PDF and CDF as in Bitcoin. For simplicity, we ignore the block propagation time among all miners. Thus, the delay only comes from the communication time between a miner and



(a) Probability density function of a conflicting block being found while there exists another block being propagated in the network [2].

(b) Average number of conflicting blocks per 24 hours as a function of communication delay, averaged over all the nodes in the network [3].

Fig. 2: Communication delay can cause occurrences of conflicting blocks.

an SP. We denote m_i's winning probability will be affected by the a delay discount function, denoting β. Given the closeness of the ESP, we only consider the miner communication delay to the CSP, denoting D_c, which will incur a discount rate of β (short for $\beta(D_c)$).

B. Expression of Individual Winning Probability

In this part, we will derive an expression of W_i under the assumption that each miner m_i's request, denoted by a vector $r_i = [e_i, c_i]^\intercal$, is fully satisfied at the SP side. Let $\boldsymbol{r} \triangleq \{r_1, r_2, \cdots, r_N\}$ and $\boldsymbol{r}_{-i}$ represent the request profile of all miners and all other miners except m_i, respectively. We denote E in Eq. (2a) and C in Eq. (2b) as the total number of computing units requested on the ESP and the CSP, respectively. $S = E + C$ therefore represents the total requested computing units in the network. The winning probability, in the form of $W_i = W_i^e + W_i^c$, consists of two parts, W_i^e and W_i^c, jointly contributed by the ESP and the CSP, where W_i^e and W_i^c are functions of r_i and $\boldsymbol{r}_{-i}$ given below:

$$W_i^e(r_i, \boldsymbol{r}_{-i}) = e_i/S + e_i \sum\nolimits_{j \neq i} \beta c_j/ES, \tag{4}$$

$$W_i^c(r_i, \boldsymbol{r}_{-i}) = c_i/S - c_i \sum\nolimits_{j \neq i} \beta e_j/ES. \tag{5}$$

For a better understanding, we begin with the analysis on W_i^c.

- W_i^c: c_i/S represents the expected chance that m_i mines a cloud-solved block b. Now we discuss the probability that b is discarded before it reaches consensus. With a chance of β, a conflicting block b' would be found during the propagation time D_c. A cloud-solved b' has the same propagation time D_c and thus cannot beat b. However, b will be discarded if b' is found in the edge and hence reaches consensus immediately. W_i^c in Eq. (5) characterizes the fact that, the probability of a successful cloud mining is discounted by the chance that the mined block is discarded due to any conflicting edge-solved block. Here, e_j/E approximates the rate that b' is mined in the edge by another miner m_k. We don't consider the situation, where b' is an edge-solved block for m_i itself, as a discount factor, since m_i still wins.
- W_i^e: m_i's winning probability of edge mining is the addition of (i) the chance that m_i is the first to successfully mine a block using its edge computing power, expressed as e_i/S and (ii) a summed chance that m_i's edge-solved block surpasses a cloud-solved block mined by any other miner m_k. The corresponding expression is shown in Eq. (4).

We verify the validity of W_i as a probability mass function.

Theorem 1. *$W_i = W_i^e + W_i^c$ is a valid probability mass function to express the winning probability of individual miners in a mobile blockchain mining network.*

Proof. We present the full verification process by checking that $\sum_{i=1}^N W_i = 1$ holds.

$$\begin{aligned}\sum\nolimits_{i=1}^N W_i &= \sum\nolimits_{i=1}^N (W_i^e + W_i^c) \\ &= \sum\nolimits_{i=1}^N [e_i/S + c_i/S] \\ &\quad + \beta \sum\nolimits_{i=1}^N [e_i(C - c_i)/ES + c_i(E - e_i)/ES] \\ &= 1 + \beta \sum\nolimits_{i=1}^N (e_i C - c_i E)/ES = 1. \qquad \square\end{aligned}$$

Thus, we are now ready to conclude that, the winning probability we use is valid, hence our model as well. Note that m_i's winning probability and hence its utility depends not only on its own request but also on the other miners'.

C. User Requests and SP Responses

All above analysis is based on the assumption that m_i's request r_i is responded to by the ESP and the CSP as what it expects, *i.e.*, if r_i is fully satisfied by the ESP and the CSP as its original form $[e_i, c_i]^\intercal$ (indicating the edge request hits the ESP's capability), the individual winning probability here is denoted by W_i^h (h means hit) and shown in Eq. (6):

$$W_i^h = (e_i + c_i)/S + \beta(e_i C - c_i E)/ES. \tag{6}$$

However, this assumption cannot always hold when we take the ESP's computing capability into consideration. It is possible that overall requests from the miner side are beyond the ESP's computing capability. Thus, we further refine the individual winning probability based on whether e_i can be satisfied by the ESP or not. Now we discuss how r_i will be responded to if e_i is beyond the ESP's capability in the both modes. We denote the corresponding winning probability by W_i^{1-h} ($1-h$ means miss).

1) Failure in connected mode: In this case, e_i would be transferred from the ESP to the CSP, and therefore, r_i is degraded as $[0, e_i + c_i]^\intercal$. The total computing power in the network stays unchanged as S, while $E - e_i$ and $C + e_i$ represent the actual resource allocation by the ESP and the CSP, respectively. Eq. (7) gives the winning probability.

$$W_i^{1-h} = (1 - \beta)(e_i + c_i)/S. \tag{7}$$

2) Failure in standalone mode: Since e_i would be rejected by the ESP, r_i is degraded as $[0, c_i]^\intercal$. Thus, the total computing power of edge computing and that in the network are reduced to $E - e_i$ and $S - e_i$, respectively. Eq. (8) gives the corresponding winning probability.

$$W_i^{1-h} = (1 - \beta)c_i/(S - e_i). \tag{8}$$

IV. Fixed Miner Number Scenario

In the fixed miner number scenario, we assume that the network contains a fixed set of miners. That is, the number of miners is modeled as a constant, *i.e.*, $N \triangleq n$. We consider two edge computing operation modes: connected and standalone. We apply backward induction to analyze the subgame perfect NE in each stage for both modes. In the connected mode, the uniqueness of the SE is validated by identifying the best response strategies of the miners. In the standalone mode, the existence of the SE is proved by capitalizing on the variational inequality theory. Then, we get the closed-form price and resource allocation solutions to a special homogeneous-miner case for both modes. Besides, we compare the profits at the SP side and the miner side in these two modes

A. Connected Mode

In this mode, the ESP's limited computing capability is characterized by the ESP's expected transfer rate $(1-h)$.

1) Miner Subgame Equilibrium: Consequently, m_i should consider two possible results: (i) with a probability of h, its request on the ESP is satisfied; (ii) with a probability of $(1-h)$, its request on the ESP is automatically transferred to the CSP with a degraded service. Thus, W_i can reflect these two results by applying the law of total expectation as shown in Eq. (9)

$$\begin{aligned} W_i &= h \cdot W_i^h + (1-h) \cdot W_i^{1-h} \\ &= h \cdot [(e_i + c_i)/S + \beta \cdot (e_i C - c_i E)/ES] \\ &\quad + (1-h) \cdot (1-\beta)(e_i + c_i)/S \\ &= (1-\beta)(e_i + c_i)/S + \beta h e_i / E, \end{aligned} \tag{9}$$

then given the budget B_i, the OP_{MINER} problem for m_i can be concreted as below.

Problem 1a (MINER SUBGAME: $\text{NEP}_{\text{MINER}}$).

$$\textit{maximize} \quad U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i), \tag{10a}$$

$$\textit{subject to} \quad P_e \cdot e_i + P_c \cdot c_i \le B_i, \quad e_i \ge 0, \quad c_i \ge 0, \tag{10b}$$

$$\textit{where } W_i = (1-\beta)(e_i + c_i)/S + \beta h e_i / E.$$

Thus, the existence and uniqueness of an NE of this miner subgame is given by the following theorem.

Theorem 2. *A unique Nash equilibrium exists in* $\text{NEP}_{\text{MINER}}$.

Proof. Denote the equivalent variational inequality (VI) problem [4] $\mathcal{VI}(\mathcal{K}, \mathrm{F}) \equiv \mathcal{NEP}(\mathcal{X}, U)$, where

$$\begin{aligned} &\mathrm{F} := (\nabla_i U_i)_{i=1}^n, \quad \mathcal{X} = ([e_i, c_i]^\intercal)_{i=1}^n, \quad \mathcal{U} = (U_i)_{i=1}^n, \\ &\mathcal{K} := \mathcal{K}_1 \times \mathcal{K}_2 \times \cdots \times \mathcal{K}_n, \\ &\mathcal{K}_i := \{(e_i, c_i) | P_e \cdot e_i + P_c \cdot c_i \le B_i, e_i \ge 0, c_i \ge 0\}. \end{aligned} \tag{11}$$

Obviously, (i) $\mathcal{K}_i$ is closed and convex, $\forall i$ and (ii) U_i is continuously differentiable and convex w.r.t. $[e_i, c_i]^\intercal$, $\forall i$, then the VI problem has a non-empty solution set. The existence of NE thus follows the sufficient conditions. Details and the proof of its uniqueness can be found on our website. □

As a rational player, each miner optimizes its utility by solving the $\text{NEP}_{\text{MINER}}$ problem as follows. Using Lagrange's multipliers λ_1, λ_2, and λ_3 for the constraints in Eq. (1d), the optimization problem is thus converted to the form

$$\begin{aligned} L_i &= R \cdot [(1-\beta)(e_i + c_i)/S + \beta h e_i / E] - (P_e \cdot e_i + P_c \cdot c_i) \\ &\quad - \lambda_1 (P_e \cdot e_i + P_c \cdot c_i - B_i) + \lambda_2 e_i + \lambda_3 c_i, \end{aligned} \tag{12}$$

and the complementary slackness conditions are

$$\begin{aligned} &\lambda_1 (P_e \cdot e_i + P_c \cdot c_i - B_i) = 0, \\ &\lambda_2 e_i = 0, \quad \lambda_3 c_i = 0, \quad \lambda_1 > 0, \lambda_2, \lambda_3, e_i, c_i \ge 0. \end{aligned} \tag{13}$$

By the first-order optimality condition $\nabla L_i = 0$, it immediately follows that $\lambda_2 = \lambda_3 = 0$. Thus

$$\begin{aligned} e_i &= \sqrt{\frac{h\beta E_{-i} R}{(1+\lambda_1)(P_e - P_c)}} - E_{-i}, \\ c_i &= \sqrt{\frac{R(1-\beta)(E_{-i} + C_{-i})}{(1+\lambda_1) P_c}} - \sqrt{\frac{h\beta E_{-i} R}{(1+\lambda_1)(P_e - P_c)}} - C_{-i}, \\ B_i &= P_e e_i + P_c c_i, \text{where } E_{-i} = \sum\nolimits_{j \ne i} e_j, C_{-i} = \sum\nolimits_{j \ne i} c_j. \end{aligned} \tag{14}$$

Solving Eq. (14) yields that

$$1 + \lambda_1 = \left[\frac{(P_e - P_c)\sigma_1 \sqrt{E_{-i}} + P_c \sigma_2 \sqrt{E_{-i} + C_{-i}}}{B_i + P_c C_{-i} + P_e E_{-i}} \right]^2, \tag{15}$$

where: $\sigma_1^2 = h\beta R/(P_e - P_c)$ and $\sigma_2^2 = (1-\beta)R/P_c$. Hence substituting Eq. (15) back into Eq. (14) gives the explicit form of the solution to the $\text{NEP}_{\text{MINER}}$ problem, *i.e.*, each miner's best response strategy. This naturally gives a distributed iterative algorithm, allowing each miner to iteratively update its strategy, given the strategies of other miners. When this unique subgame NE is ensured, the algorithm's convergence is also guaranteed. The uniqueness of NE in $\text{NEP}_{\text{MINER}}$ is guaranteed given that F defined in Eq. (11) is strictly monotone.

2) SP Subgame Equilibrium: The ESP's and the CSP's problems can be rewritten as below:

Problem 2a (SP SUBGAME: NEP_{SP}).

$$\textit{maximize} \quad V_e = (P_e - C_e) \cdot E \textit{ where } E = \sum\nolimits_{i=1}^N e_i, \tag{16a}$$

$$\textit{maximize} \quad V_c = (P_c - C_c) \cdot C \textit{ where } C = \sum\nolimits_{i=1}^N c_i. \tag{16b}$$

With the knowledge of the miners' strategies, each SP makes its decision by solving the NEP_{SP}.

Theorem 3. *A unique Nash equilibrium exists in* NEP_{SP}.

Based on the unique Nash equilibrium achieved among all miners, each SP can optimize its strategy to achieve profit maximization. According to [5], since each SP's objective function is continuous and concave, there exists a unique Nash equilibrium among them.

3) Stackelberg Equilibrium: We take advantage of a classic distributed algorithm (Algorithm 1) called Asynchronous Best-response [6] to find the unique NE point in OP_{SP} defined in Problem 2, where an SP is engaged in a gradient ascent process to maximize its utility. The solution's uniqueness further guarantees the global convergence and SE is achieved, given that NE is found in the leader stage.

https://sites.temple.edu/suhanjiang/2019/01/12/supplementary-materials/

Algorithm 1 Best Response Algorithm

Output: $j, j \in \{e, c\}$

Input: Initialize k as 1 and randomly pick a feasible $P_j^{(0)}$

for iteration k **do**
  Receive the miners' request vectors $\mathbf{r}^{(k-1)}$
  Predict the strategy of the other SP
  Decide $P_j^{(k)} = P_j^{(k-1)} + \Delta \frac{\partial V_j\left(P_j, P_{-j}^{(k-1)}, \mathbf{r}^{(k-1)}\right)}{\partial P_j}$
  if $P_j^{(k)} = P_j^{(k-1)}$ **then** Stop
  else send $P_j^{(k)}$ to miners and set $k \leftarrow k+1$

4) Homogeneous Miners with Identical Budgets: The solutions to the $\text{NEP}_{\text{MINER}}$ are infeasible to express in a symbolic manner. Fortunately, we are able to get the closed-form computation offloading solutions for the $\text{NEP}_{\text{MINER}}$ in a special case. We consider a homogeneous-miner case where each miner is homogeneous with an identical budget B. We are interested in finding an NE where miners decide on a mixed request, buying computing units from both the ESP and the CSP. Thus, constraint (10b) is modified as $e_i > 0$, $c_i > 0$. The corresponding miner side optimization problem can be rewritten as the $\text{NEP}_{\text{HOMOMINER}}$ problem in the following.

Problem 1b (MINER SUBGAME: $\text{NEP}_{\text{HOMOMINER}}$).

$$\text{maximize} \quad U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i), \tag{17a}$$

$$\text{subject to} \quad P_e \cdot e_i + P_c \cdot c_i \le B, \quad e_i > 0, \quad c_i > 0, \tag{17b}$$

where $W_i = (e_i + c_i)/S + \beta \cdot (e_i C - c_i E)/(ES)$.

We will provide the explicit-form expression or the pricing strategy for the homogeneous-miner case defined above.

Theorem 4. *The unique Nash equilibrium for miner m_i in the $\text{NEP}_{\text{HOMOMINER}}$ problem is given below*

$$\begin{cases} e_i^* = \dfrac{B\beta h}{(1-\beta+h\beta)(P_e-P_c)}, \\ c_i^* = \dfrac{B\left[(1-\beta)(P_e-P_c) - P_c\beta h\right]}{P_c(1-\beta+h\beta)(P_e-P_c)}, \end{cases} \tag{18}$$

provided that the prices set by the ESP and the CSP satisfy $P_c < \dfrac{1-\beta}{1-\beta+h\beta} P_e$.

Proof. According to Eq. (14), we have $E^2 = \sigma_1^2 \sum_{j \ne i} e_j/(1+\lambda_1)$ and $S^2 = \sigma_2^2 \sum_{j \ne i} (e_j + c_j)/(1+\lambda_1)$ for each miner m_i, which will yield $E^2/S^2 = \sigma_1^2(E-e_i)/[\sigma_2^2(S-e_i-c_i)]$. Then, we calculate the summation of this expression for all the miners: $E/S = \sigma_1^2/\sigma_2^2 = [h\beta/(1-\beta)] \cdot P_c/(P_e-P_c)$. In order to get a mixed strategy, $E/S > 1$ must hold, *i.e.*, Eq.(4) holds. Since all miners are homogeneous, their best response strategies are identical as well, *i.e.*, $E = Ne_i$ and $S = N(e_i + c_i)$. By substituting these two equations into Eq. (15), we obtain the NE for miner m_i in Eq.(18). □

Corollary 1. *If the budget B is sufficiently large, the explicit solution to the $\text{NEP}_{\text{HOMOMINER}}$ problem is shown in Eq.(19)*

$$\begin{cases} e_i^* = \dfrac{\beta h R(N-1)}{N^2(P_e-P_c)}, \\ c_i^* = \dfrac{R(N-1)\left[(1-\beta)P_e - P_c\right]}{N^2 P_c(P_e-P_c)}. \end{cases} \tag{19}$$

Now, we start to analyze the SP optimization problem, which can be rewritten as follows.

Problem 2b (SP SUBGAME: $\text{NEP}_{\text{SPHOMOMINER}}$).

$$\text{maximize} \quad V_e = (P_e - C_e)Ne_i^*, \; V_c = (P_c - C_c)Nc_i^*, \tag{20a}$$

$$\text{subject to} \quad P_c < \frac{1-\beta}{1-(1-h)\beta} P_e, \tag{20b}$$

$$\text{where } e_i^* = \frac{B\beta h}{(1-\beta+h\beta)(P_e-P_c)}, \; c_i^* = \frac{B\left[(1-\beta)(P_e-P_c)-P_c\beta h\right]}{P_c(1-\beta+h\beta)(P_e-P_c)}.$$

Theorem 5. *The unique Nash equilibrium for the SPs in the $\text{NEP}_{\text{SPHOMOMINER}}$ problem is given below:*

$$\begin{cases} P_e^* = \bar{p}, \\ P_c^* = \dfrac{C_c\bar{p}(1-\beta) - \bar{p}\sqrt{C_c h\beta(\bar{p}-C_c)(1-\beta)}}{[1-\beta(1-h)]C_c - \beta h P_e}, \end{cases} \tag{21}$$

where $\bar{p}$ is the solution to $\partial V_e/\partial P_e = 0$.

Proof. We start with the optimal P_c^* by analyzing the convexity of V_c. We calculate the first derivative of V_c and find that it is a concave function. Thus, the CSP's optimal price value is the solution to $\partial V_c/\partial P_c = 0$ where $P_c < P_e(1-\beta)/[1-(1-h)\beta]$ and P_c^* is shown in Eq. (21), as is a function dependent on P_e set by the ESP. Given the response strategy of the CSP, the ESP can optimize his payoff by maximizing the re-written V_e, which is given below:

$$V_e = \frac{NB\beta h}{(1-\beta+h\beta)(P_e-P_c^*)} \cdot (P_e - C_e). \tag{22}$$

We calculate the second derivative of V_e and find that $\partial^2 V_e/\partial P_e^2 \le 0$ holds for any valid P_e value. Thus, the ESP has his dominant strategy $P_e^* = \bar{p}$. In this situation, NE is achieved in the leader stage. We analyze P_e^* and P_c^* and find that they only depend on their own operating costs C_e, C_c, and the network delay penalty factor β. □

B. Standalone Mode

In standalone mode, the ESP only has a total of E_{max} computing units, where E_{max} is a common knowledge in this game. It has to reject some requests when overloaded. Thus, the aggregate requests from all miners should be no more than E_{max} in order to avoid being rejected.

1) Subgame Equilibrium: In standalone mode, given other miners' requests $\boldsymbol{r}_{-i}$, m_i should ensure that e_i can be satisfied by the ESP. Mathematically, this can be written as $E = \sum_{k=1}^{n} e_k \le E_{max}$. Under this constraint, its winning probability is expressed in Eq. (23).

$$W_i = (e_i + c_i)/S + \beta(e_i C - c_i E)/ES. \tag{23}$$

Now, we reformulate the OP_{MINER} problem in the following.

Algorithm 2 Price Bargaining

Input: Choose any feasible starting point P_e, P_c

for each miner i **do**
  Receive P_e, P_c
  Predict the optimal requests of other miners
  Decide its computing request $[e_i, c_i]^T$
  Send e_i to the ESP and send c_i to the CSP
for each operator j, $j \in \{e, c\}$ **do**
  Receive the optimal requests of miners
  Store the current prices $P_j^{'}$ and $P_{-j}^{'}$,
  Increase and decrease the price with a small step Δ
  if $V_j\left(P_j^{'}, P_{-j}^{'}\right) \leq V_j\left(P_j^{'}+\Delta, P_{-j}^{'}\right)$ and
    $V_j\left(P_j^{'}-\Delta, P_{-j}^{'}\right) \leq V_j\left(P_j^{'}+\Delta, P_{-j}^{'}\right)$
  then $P_j = P_j^{'} + \Delta$
  else if $V_j\left(P_j^{'}, P_{-j}^{'}\right) \leq V_j\left(P_j^{'}-\Delta, P_{-j}^{'}\right)$ and
    $V_j\left(P_j^{'}+\Delta, P_{-j}^{'}\right) \leq V_j\left(P_j^{'} - \Delta, P_{-j}^{'}\right)$
  then $P_j = P_j^{'} - \Delta$
  else $P_j = P_j^{'}$
  Send P_j to miners

Problem 1c (MINER SUBGAME: GNEP$_{\text{MINER}}$).

$$\begin{aligned} \textit{maximize} \quad & U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i), && \text{(24a)} \\ \textit{subject to} \quad & E \leq E_{max}, && \text{(24b)} \\ & P_e \cdot e_i + P_c \cdot c_i \leq B_i, \quad e_i, c_i \geq 0, && \text{(24c)} \end{aligned}$$

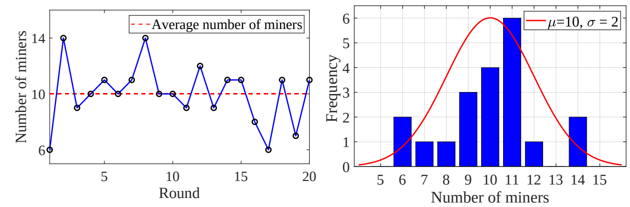
where $W_i = (e_i + c_i)/S + \beta \cdot (e_i C - c_i E)/ES$. Constraint (24b) ensures that m_i's request to the ESP can be satisfied.

Since all miners' requests are mutually dependent, the GNEP$_{\text{MINER}}$ problem is a Generalized Nash Equilibrium Problem (GNEP). In GNEP$_{\text{MINER}}$, the dependence of each miner's strategy set on the other miners' strategies is represented by the (linear) constraint (24b), which includes each miners' request e_i to the ESP. More specifically, since the miners all share a jointly convex shared constraint (JCSC), this subgame is known as a jointly convex game.

2) Existence of Stackelberg equilibria: Similar with the proof for NEP$_{\text{MINER}}$ NE in Theorem 3, the existence of GNEP$_{\text{MINER}}$ NE is easily followed by capitalizing on the variational inequality theory.

Theorem 6. *Given a price set $\{P_e,\ P_c\}$ from the SP side, there exists at least one Nash equilibrium for the non-cooperative subgame at miner side in standalone mode.*

In general, a GNEP could have infinite solutions. Namely, there are multiple NEs in the follower stage, and thus there is no efficient algorithm to obtain the global optimal pricing and computation offloading strategy for the proposed Stackelberg game. Here, we provide a distributed algorithm which first computes a unique variational solution to the GNEP$_{\text{MINER}}$ problem and then finds the corresponding solution to the SP SUBGAME: GNEP$_{\text{SP}}$ problem (defined in the below) based on the computed miner Nash equilibrium.



(a) Statistics on the miner number among 20 mining rounds. (b) Corresponding histogram and underlying distribution $N(\mu, \sigma^2)$.

Fig. 3: A toy example for population dynamics of moblie miners.

TABLE II: Optimal requests of homogeneous miners with sufficiently large budgets where $\gamma = (N-1)R/N$.

Mode	E^*	C^*	S^*
Connected	$\frac{\gamma\beta}{P_e - P_c} h$	$\gamma\left[\frac{(1-\beta)P_e - P_c}{P_c(P_e - P_c)} + \frac{\beta(1-h)}{P_e - P_c}\right]$	$\frac{\gamma(1-\beta)}{P_c}$
Standalone	$\frac{\gamma\beta}{P_e - P_c}$	$\gamma\frac{(1-\beta)P_e - P_c}{P_c(P_e - P_c)}$	$\frac{\gamma(1-\beta)}{P_c}$

Problem 2c (SP SUBGAME: GNEP$_{\text{SP}}$).

$$\begin{aligned} \textit{maximize} \quad & V_e = (P_e - C_e) \cdot E, \quad V_c = (P_c - C_c) \cdot C, && \text{(25a)} \\ \textit{subject to} \quad & E = E_{max}. && \text{(25b)} \end{aligned}$$

Both SPs can achieve their own equilibrium price by applying Algorithm 2, where they only adjust the price towards a utility improvement direction. Note, there is no guarantee that the produced SE is a global optima.

3) Homogeneous Miners with Sufficiently Large Budgets: In the standalone mode, we also consider a special case where all miners are considered with a large amount of budgets in that their optimization problem is never constrained by the lack of sufficient budgets. Since the analysis is quite similar as is in the connected mode, we just list our results in Table II, which is compared to the corresponding results in the connected mode. From Table II, we find that the total requested units at the miners' side remain unchanged, while the standalone mode encourages more purchases from the ESP. The numerical results provided in Section VI also show that the ESP's equilibrium prices in the standalone mode is higher compared to those in the connected mode. Thus, we conclude that the ESP in the standalone mode gains more profits if miners have large budgets. However, the explicit solution for small budget cases is still an open problem in the standalone mode.

V. DYNAMIC MINER NUMBER SCENARIO

Obviously, in the above analysis, we assume the miner number N is common knowledge in the proposed games. In practice, this scenario is applicable to permissioned blockchains, where miners are pre-selected by a central authority or consortium. However, most blockchains are permissionless, in which anyone can participate in or retreat from the mining process, so the previous scenario may not be suitable. For such situations, we consider a more general scenario by introducing population uncertainty. Games with population uncertainty relax the assumption that the exact number of players is common knowledge. Thus, we model the miner number, N, as a random variable. In particular, N follows a Gaussian distribution with mean μ and variance σ^2. We have $N \sim \mathcal{N}(\mu, \sigma^2)$ where $N = k$ with probability $P(k) = \Phi(k) - \Phi(k-1)$. Fig. 3 gives a toy example where the number of miner can be fit to a Gaussian distribution with $\mu = 10$ and $\sigma^2 = 4$.

In this scenario, we only consider standalone mode and derive the miner utility function U_i as below.

$$U_i(\mu, \sigma^2) = 0.5 \cdot U_i^h + 0.5 \cdot U_i^{1-h} \tag{26}$$
$$U_i^h = P_e \cdot e_i + P_c \cdot c_i - R \cdot W_i^h$$
$$U_i^{1-h} = P_e \cdot e_i + P_c \cdot c_i - R \cdot W_i^{1-h}$$
$$W_i^h = \sum_{k=l}^{u} P(k) \left[(e_i + c_i)/S_k + \beta(e_i C_k - c_i E_k)/(S_k E_k)\right]$$
$$W_i^{1-h} = (1-\beta)(e_i + c_i)/S_\mu$$
$$S_k = E_k + C_k, E_k = \sum_{j=1}^{k} e_j, C_k = \sum_{j=1}^{k} c_j, \forall k \in [l,\, u]$$

Thus, the OP$_{\text{MINER}}$ problem in this scenario can be reformulated as in Eq. (27).

Problem 1d (MINER SUBGAME: OP$_{\text{DYNAMICMINER}}$).

$$\textit{maximize} \quad U_i(\mu, \sigma^2) \tag{27a}$$
$$\textit{subject to} \quad P_e \cdot e_i + P_c \cdot c_i \leq B_i, \quad e_i \geq 0, \quad c_i \geq 0 \tag{27b}$$

Problem 2d (SP SUBGAME: OP$_{\text{SP}}$).

$$\textit{maximize} \quad V_e = (P_e - C_e) \cdot E, \;\; V_c = (P_c - C_c) \cdot C \tag{28}$$

The objective function presented in Eq. (27) is so complex that it is infeasible to express its equilibrium expression in a symbolic manner. However, we still can find equilibria in the network by utilizing a modified version of Algorithm 2 (where miners will apply the new objective function). As experimental results will later show in Section VI, we find that with an identical P_e, the uncertainty incurred by the dynamic population renders miners more aggressive to buy computing units from the ESP, even beyond its capability E_{max}.

VI. Simulation

A. Miner Subgame Equilibrium

We start with a small mobile blockchain mining network with only 5 miners with budgets B_i, $\forall i \in [1, 5]$. Our experiments evaluate how the corresponding miner subgame Nash equilibrium is influenced if the parameter values change.

1) Influences from SP side: We first consider the different prices at SP side. Assuming a homogeneous-miner case in the connected mode, where $B_i = 200$, $\forall i \in [1, 5]$ holds, Fig. 4 obviously reflects that, if the CSP's price P_c unilaterally increases, miners tend to buy more units from the ESP, leading to more revenue at the ESP side. Similarly, from Fig. 4, we can also conclude that the blockchain fork rate β caused by the CSP's communication delay also has a negative effect on the number of total units sold by the CSP as well as his total revenue. However, from Fig. 5(c), we find the total revenue at the SP side remains almost unchanged no matter how prices and communication delay change. In the same miner configuration, we analyze the impact of edge operation modes. If the ESP operates in the standalone mode, we see its computing capability is positively related to miners' requests, which can be easily followed in Fig. 6. From this figure, we can conclude that, miners are discouraged from buying units from an ESP working in the connected mode. We see a cross in the Fig. 6. This explains the CSP's optimal prices under different communication delays. The longer the communication delay, the lower the optimal price.

2) Influences at miner side: Miners also mutually affect each other in this mining network. Fig. 7 shows the changes on all the miners' utilities when their budget of B_i varies from 20 to 200. m_i's requests to the ESP and the CSP keep increasing and its utility follows a similar trend. However, we can see that m_i's total requests to both SPs are similar even with different communication delays at the CSP side.

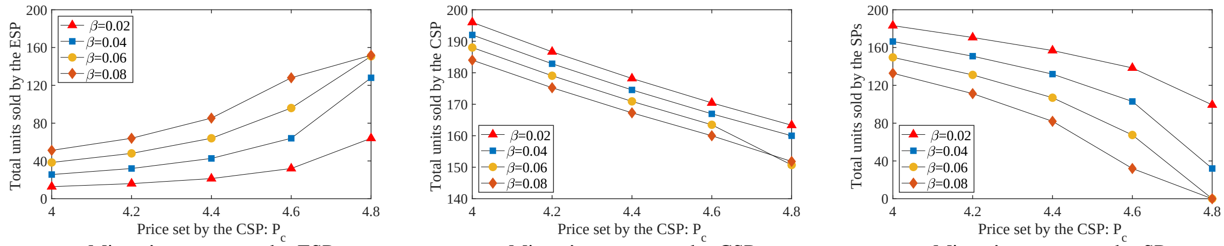
B. SP Subgame

We also study how communication delay and edge operation modes as well as the SP's operating costs affect their equilibrium prices. Fig. 8 depicts the equilibrium prices of the SPs. The ESP's prices increase linearly as its unit operating cost increases. In both modes, the ESP charges a higher price, because it has less power available and its communication delay is shorter in the proposed network. However, its advantage will be shaded if the communication delay at the CSP side decreases. Besides, the ESP's computation limitation also poses an upper bound on its profits. We also discover that the standalone mode allows the ESP a higher price while it decreases the CSP's price and its profits.

According to numerous experiments, we find that the total amounts of sold-out computing units are always approximately equal, if we allow a sufficiently large budget and a fixed number of miners. Besides, we can see that the SP-side welfare is bounded by the total miner budgets in the beginning. However, as the budgets increase to a certain degree, the total welfare of these two SPs are positively related to the blockchain mining reward.

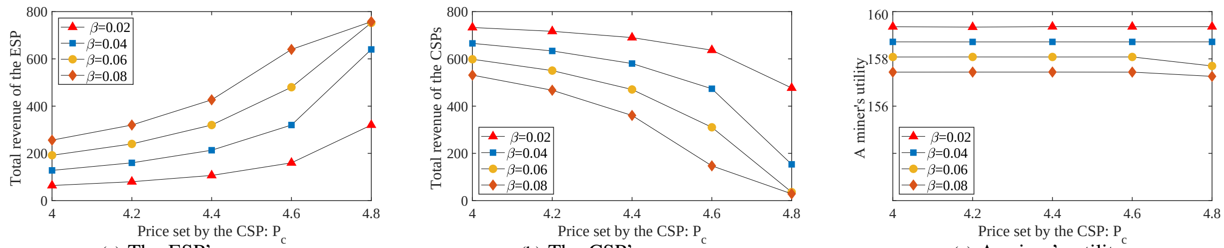
C. Population Uncertainty

In Section V, we consider the miner number as a variable subject to a specific Gaussian distribution. To capture the dynamics of the miner number, we use a reinforcement learning (RL) framework to allow miners to learn the population uncertainty and hence improve their strategies. We conduct our simulation within a small mining network of 5 homogeneous miners. We define a time period T as adding 50 blocks. During T, prices from these two SPs are fixed and the miner number changes subject to $N(\mu, \sigma^2)$. The reason why we choose $T = 50$ in our all experiments is that miners' strategies converge after at most 50 blocks added even in such an unstable-population mining network. Once the miners' behavior converges, both the ESP and the CSP update their pricing strategies adaptively. These two steps repeat until a fixed point for both sides is reached. We also apply such a process to a fixed number scenario where $N = \mu$.



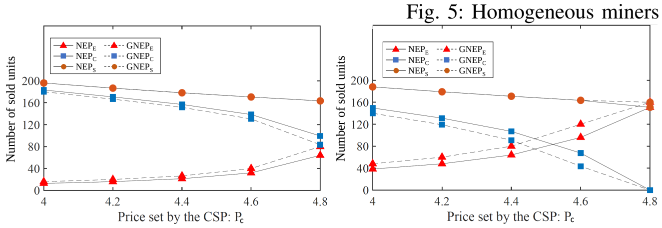
(a) Miners' requests to the ESP. (b) Miners' requests to the CSP. (c) Miners' requests to the SPs

Fig. 4: Homogeneous miners with identical budgets and $P_e = 5$.



(a) The ESP's revenue. (b) The CSP's revenue. (c) A miner's utility.

Fig. 5: Homogeneous miners with identical budgets and $P_e = 5$.



(a) $\beta = 0.02$. (b) $\beta = 0.06$.

Fig. 6: Connected Vs Standalone.

In Fig. 9, all unfilled points are the results produced by the RL framework, while all lines are computed using our proposed model. The results of our model are anastomotic with the learned strategies. In Fig. 9(a), we conclude that the uncertainty caused by the miner number renders each miner to buy more units from the ESP, making the total requests sometimes can exceed the ESP's capability. Besides, we also find the variance also affects a miner's request to the ESP, *i.e.*, a larger variance leads to a more ESP-prone miner, according to Fig. 9(b), where $N(5, 0.25)$ represents a normal distribution of which the mean is 5 and the variance is 0.25.

VII. Related Work

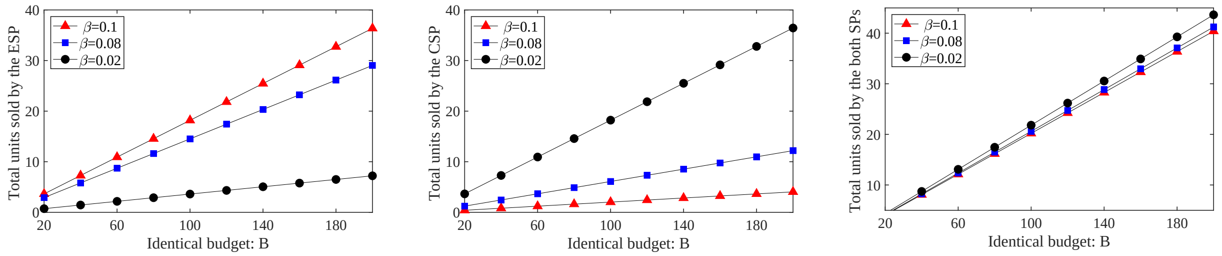
1) Mobile Blockchain Applications: There exist two different categories of research in the field of blockchain applications in wireless networks. The first category focuses on blockchain protocols [7, 8] to eliminate overhead while maintaining most of blockchain's security and privacy. These research works are beneficial for secure and decentralized data communication in wireless networks. Instead of designing and implementing light-weight blockchain-based protocols, the second category [9–12] investigates pricing and resource management schemes for supporting blockchain applications in a mobile environment. The focus here is on the mining under the PoW consensus [1], which results in the competition among miners to receive a mining reward. Due to limited computing resources of their mobile terminals, miners offload the PoW computations to local edge servers [9, 10]. In this paper, we also study the problem of pricing and computation offloading in mobile blockchain mining under the PoW consensus. However, we consider a more complicated assumption in which miners can perform a two-layer computation offloading to either/both of the ESP and the CSP.

2) Stackelberg Game in Offloading Mechanism: Stackelberg Game is a widely-used model in the field of offloading mechanisms. A large body of existing literature [13–20] focuses on minimizing offloading users' computation overhead in terms of energy and latency. To this end, researchers have developed distributed decision making methodologies. In the field of mobile blockchain mining offloading [9, 10], there are few works and most of them are in the single-leader scenario where mobile miners only offload their computation to an SP, *e.g.* fog. In our paper, we consider a multi-leader multi-follower Stackelberg game to jointly maximize the profit of the SPs and the individual utilities of mobile miners. We assume a resource-limited edge layer working in either stand-alone or connected operation mode with the cloud layer.

3) Reinforcement Learning in Incomplete Information Game: Although analysis in game theory always assumes the observable strategies of other players, it is more realistic that the miner's action is the private information which is unobservable by others. In addition to applying game-theoretical analysis on the proposed game, we also develop a reinforcement learning framework [21–24] in our evaluation, allowing all players to select their best response strategies and update their beliefs about unobservable actions of others through repeated interactions with each other in a stochastic environment. This framework confirms our proposed model.

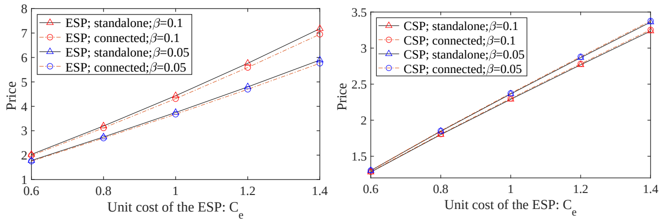
VIII. Conclusion

In this paper, we have proposed a Stackelberg game between the SPs for optimal price strategies and among the mobile miners for optimal computation offloading requests. Two practical edge computing operation modes are investigated, *i.e.*, the ESP is connected to the CSP or standalone. First, we characterize



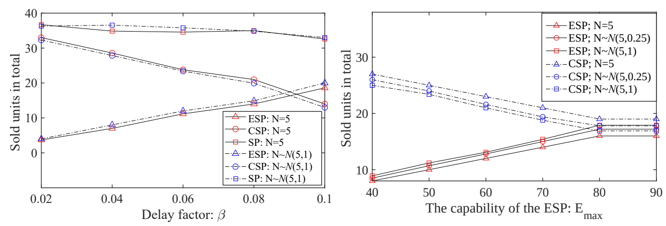
(a) A miner's request to the ESP. (b) A miner's request to the CSP. (c) A miner's request to the both SPs.

Fig. 7: m_i's budget B_1 varies from 20 to 200, with 5 miners in total.



(a) $\beta = 0.02$. (b) $\beta = 0.06$.

Fig. 8: The CSP's unit cost is 0.5, while the ESP's unit cost changes.



(a) $E_{max} = 15$. (b) $\beta = 0.06$.

Fig. 9: Miner number: fixed vs dynamic.

the miner number as a constant in both modes. We discuss the existence and the uniqueness of Stackelberg equilibrium in the proposed games and provide algorithms to achieve SE point(s). Our analysis indicates that the connected mode discourages miners from buying computing resources from the ESP. Then, we study the impact of a dynamic miner number. Interestingly, we find that uncertainty incurred by the dynamic population renders miners more aggressive to buy computing resources from the ESP. Numerical experiments based on a reinforcement learning framework have been conducted to further confirm our analysis.

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