Mobile Charger Coverage Problem for Specific Heterogeneous Wireless Sensor Networks

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***Abstract*—One of the main hindrances that wireless sensor networks (WSNs) face is the battery-powered sensors that need to be charged from time to time. Recently, the approach of having mobile chargers (MCs) that travel to the static sensors of the network and transfer energy wirelessly and efficiently has become a promising solution to that hindrance. An optimization problem, called the mobile charger coverage problem, arises naturally to keep all of the sensors alive with an objective of determining both the minimum number of MCs required to meet the sensor recharge frequency and the schedule of these MCs. It is shown that this optimization problem becomes NP-hard in a high-dimensional space, including 2-D space with homogeneous recharging frequency. On the other hand, it is shown that a polynomial-time algorithm exists for sensors with a homogeneous recharge frequency on a 1-dimensional space (line or ring). In this paper, we seek to find a delicate border between the tractable and intractable problem space. Specifically, we study the special case of heterogeneous sensors that take frequencies of 1's and 2's (lifetimes of 1 and 0.5 time units) on a line, conjecture its NP-hardness, propose a novel brute-force optimal algorithm, and present a linear-time greedy algorithm that gives a 1.5-approximation solution for the problem. A comprehensive simulation is conducted to verify the efficiency of using our proposed algorithms.**

***Index Terms*—Cooperative charging, linear networks, mobile chargers, wireless charging, wireless sensor networks.**

I. Introduction

There are various applications for wireless sensor networks (WSNs) with *mobile chargers* (MCs) assigned to visit the sensors in the network at certain frequencies. With the development of wireless energy charging technologies, using MCs to keep charging the sensors becomes more practical. A natural problem called *mobile charger coverage problem* arises as an optimization problem with the objective of minimizing both the number of MCs assigned to the sensors in the network and the schedule of these MCs so that all requirements are satisfied under some constraints.

In this paper, we study the mobile charger coverage problem for a 1-dimensional (1-D) line specific heterogeneous WSN, construct an optimal solution, and propose an approximation algorithm for it. Each sensor needs to be visited by an MC on a specific frequency. These MCs are assumed to recharge the sensors instantly by visiting their location. Furthermore, the MCs' speed is limited to a certain maximum speed, and they have an unlimited charging capability.

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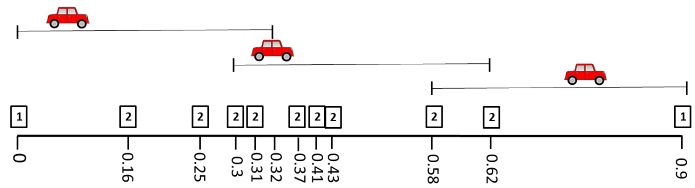

Fig. 1: Toy example for the problem showing an optimal MC-solution.

Formalizing the general mobile charger coverage problem, we consider a multi-dimensional space with a distribution of sensor nodes $S = \{s_i\}$ with an assigned fixed location x_i for every sensor s_i. For simplicity, each sensor s_i will be denoted by its location x_i. Each one of these sensor nodes x_i needs to be visited by an MC node from the set of deployed $MC = \{MC_i\}$ at a given frequency f_i, i.e. x_i has to be visited by one of the MCs no more than $1/f_i$ after the previous visit occurred at x_i. An optimization problem arises to determine the minimum number of MCs needed to satisfy the charging requirement of the sensors, the MCs' coverage areas, and their velocities at every moment. A homogeneous mobile charger coverage problem is the problem where the frequencies are equal for all of the sensors. Other mobile charger coverage problems are called heterogeneous mobile charger coverage problems.

Figure 1 shows an example of a heterogeneous WSN problem with allowed frequencies of 1's and 2's. The sensors of frequency 1 and frequency 2 are denoted as boxed 1's and 2's. We will call these sensors 1-sensors and 2-sensors, respectively. In the example, we see an optimal solution for a linear WSN of eleven sensors distributed at the locations shown in the figure.

Wu *et al.* [1] have come up with an optimal solution for the homogeneous mobile charger coverage problem for both the 1-D ring and 1-D line distributions of sensors. Furthermore, they showed that the solution for a line distribution has at most one MC more than the number of MCs in the solution of the same distribution on a ring. Hence, we focus our efforts in this work to consider 1-D line distributions of sensors. Their optimal solution to solve the homogeneous line problem is done by simply scheduling k MCs to cover non-overlapping fixed intervals of length 0.5 so that all of the sensors are covered, assuming, without loss of generality, the maximum speed of the MCs to be one unit distance per unit, and the frequencies of the sensors to be 1. In addition to that, they have started the investigation of the heterogeneous problem by proposing an approximation algorithm with a factor of 2 that solves the problem for a line distribution of sensors with frequencies $f_i \in \{1, 2, \ldots, k\}$ by greedily assigning MCs with non-overlapping coverage areas that go back and forth as far as possible at maximum speed while completely supplying the demand of all the sensors in their coverage areas.

Here, we raise a concern about the delicate border in the mobile charger coverage problem between the intractable and tractable solution for it and try to fill this gap by studying the 1-D linear heterogeneous WSN problem with frequencies of 1's and 2's, we will call this heterogeneous distribution of sensors $(\mathit{1}, \mathit{2})$-WSN. For general frequencies, we can group frequencies bounded by the frequency 2^i $(i = 0, 1, 2, ...)$. That is, frequencies more than 2^i but no more than 2^{i+1} belong to virtual independent network i as discussed in [2]

Our results are summarized as follows:

- An optimal solution for $(\mathit{1}, \mathit{2})$-WSNs' mobile charger coverage problem. This solution exhausts a set of solutions with specific properties and chooses the optimal one from them. Also, we conjecture the NP-hardness of this problem.
- An approximation solution with an improved approximation ratio of 1.5 for $(\mathit{1}, \mathit{2})$-WSNs, an enhancement to this solution, and an analytical extension for the previous approximation solution.
- A comprehensive simulation to verify the closeness of our approximation solutions to the optimal one in different distributions of sensors. The distributions were chosen to model different real-life scenarios.

The remainder of the paper is organized as follows. In Section II, some related works are reviewed. In Section III, the optimal solution for the $(\mathit{1}, \mathit{2})$-WSN problem is proposed, and the NP-hardness of the problem is conjectured. In Section IV, a greedy algorithm with an approximation ratio of 1.5 for the $(\mathit{1}, \mathit{2})$-WSN is proposed, an enhancement for this solution is demonstrated, and an analytical expansion for the previously proposed 2-approximation general algorithm is performed. In Section V, simulation results are presented to compare the different proposed solutions. Finally, Section VI gives the conclusion.

II. Related Work

The breakthrough of the employment of strong magnetic resonances in wireless energy transfer technology [3] gave a reliable way to provide the sensors in WSNs with power [4]. The wireless energy transfer technology has many commercial applications [5]. Research has been conducted on wireless energy charging by applying MCs to charge sensors in WSNs [6]. Wu *et al*. [7] have formulated the mobile charging problem which allows cooperative charging of sensors by MCs in a way that guarantees none of the sensors will eventually run out of energy, which is the same constraint we have in our paper.

The same problem has been formulated with many variable parameters: considering the MCs with limited energy capacity or with unlimited energy capacity so that the MCs themselves need to be recharged periodically [1, 8, 9], considering the demand of the sensors to be deadline-based or frequency-based [10, 11], and considering the charging of the sensors to be instant once they are visited or gradual in which a charging time is needed [12, 13]. In our model, we consider the MCs to have an infinite amount of energy, charging instantly once they visit the sensors that demand their chargings on a frequency base. We assume the charging time takes zero time. If the actual charging time takes t units of time, it can be converted to our proposed model by adding distance based on the maximum velocity of the charging unit as discussed in [13].

The objective function to be optimized has some variances in the literature too. Some studied the problem trying to minimize the total distance a constant number of MCs travel [14]; others tried to minimize the maximum distance traveled by any one of the MCs [15]; while others studied minimizing the total power consumed [7], and others studied the case in which maximizing the charging throughput itself is concerned [16] . In our paper, our objective is to minimize the number of MCs needed to keep the sensors alive, similar to the model Wu *et al*. have studied [7].

Even though we consider an instant full charging of the sensors as some did [11], some have considered the problem of charging the sensors to a partial capacity with a charging rate constraint [17]. Finally, it is worth mentioning that even though 1-D [1], 2-D [18–20], and 3-D [21] WSNs have been studied for this problem, our work of studying the 1-D case remains novel since we try to investigate the NP-hardness boundary of the problem. For multi-dimensional instances of sensor distributions, we can use various dimension reduction processes, e.g., from 2-D to 1-D by constructing a spanning tree and then finding a Hamiltonian path around the tree.

III. Optimal Solution for $(\mathit{1}, \mathit{2})$-WSN

We approach the $(\mathit{1}, \mathit{2})$-WSN problem assuming the maximum speed of MCs to be one unit distance per unit time.

A. Subspace reduction

In this subsection, we reduce the search space for the MC-solution into one that has at least one optimal solution by imposing some restrictions for the possible solution. We will call our target optimal solution $\mathcal{O}$. The restrictions on the search space are:

- In the optimal solution $\mathcal{O}$, the trajectories of the MCs are end-to-end. This property holds after reducing any optimal solution to $\mathcal{O}$. This reduction can be made by replacing any detour by an extension of the MC's trajectory to one side of its coverage area. A detour is when an MC traverses from point A to point B in a time more than the minimum possible time such that the MC does not visit point B more than once nor reach the end of its coverage area.

  To prove this, we call the difference in time between the detour and the minimum possible time τ. By replacing this detour by a maximum-speed trajectory and traversing

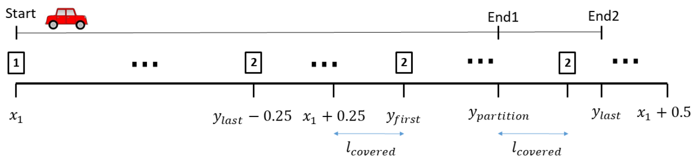

Fig. 2: Deploying an MC with no previously 'visited' sensors.

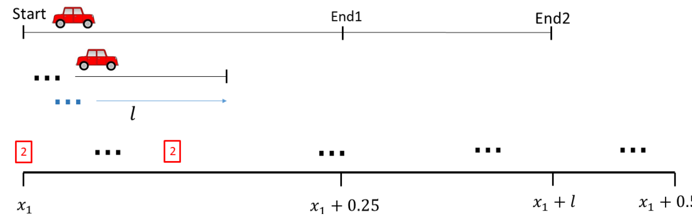

Fig. 3: Deploying an MC with previously visited sensors.

by τ time after one of the edges of the coverage area without detouring nor violating the frequency requirement of any sensor, an equivalent optimal solution will be obtained from the previous one.
- The MCs never go to the left of the leftmost sensor in $\mathcal{O}$; if there is an optimal solution in which the MCs cover an area to the left of the leftmost sensor, it can be reduced to another optimal solution in which the trajectories of the MCs are bounded from the left by the leftmost sensor.
- The MCs never meet each other. First, we perform a reduction in which the MCs never pass each other. This reduction can be made simply by swapping the velocities (direction and speed) of any two MCs passing each other. After that, if any two MCs meet, we simply apply a shift in time to the trajectory of one of them so that they do not meet.

Applying these restrictions to our search space guarantees that there will be at least one optimal solution $\mathcal{O}$ in the reduced search space. The optimal solution $\mathcal{O}$ has the following properties:

Property 1: The optimal solution $\mathcal{O}$ has the leftmost uncovered sensor completely supplied by exactly one MC.

This is a direct corollary from reducing any optimal solution to one ($\mathcal{O}$) in which MCs never meet under the imposed restrictions. Hence, we can not make two MCs supply the leftmost uncovered sensor without meeting.

Property 2: No sensor is supplied by more than two MCs in the optimal solution $\mathcal{O}$.

Since every MC will be deployed mainly to fully supply the demand of the leftmost sensor of the remaining distribution of sensors, the maximum length of any coverage area of any MC will not exceed 0.5, which requires supplying every sensor in the coverage area with energy at least once every unit of time. This means we would not need more than two MCs to supply any sensor of frequency 1 or 2.

Property 3: An MC's starting point is always more than 0.25 away from the starting point of the previous MC in $\mathcal{O}$.

We know from property 1 and property 2 that the optimal solution $\mathcal{O}$ has all of the sensors in the first 0.5 distance units completely supplied by at most two MCs. Thus, alleviating the resulting problem as much as possible will be achieved by making the next MC *able to* reach as distant away as possible away, and that only happens if the starting point of the coverage area of the next MC is after at least 0.25 distance units of the starting point of the coverage area of the previous MC. This means that the next MC will not visit the sensors in the first 0.25 distance units.

B. Algorithm overview

In this subsection, we show the high-level of an algorithm that searches for all the solutions in the reduced search space with the restrictions, then picks the one of them that uses the least number of MCs, which is $\mathcal{O}$.

Constructing all of the solutions with these properties for our $(1, 2)$-WSN, where the leftmost uncovered sensor location is x_1, will be as follows: First, deploy an MC that completely supplies the sensors in $[x_1, x_1 + 0.25]$. Completely supplying them directly implies that **1)** the start point of the coverage area of the MC is x_1, and that **2)** the endpoint of the coverage area of the MC is in $[x_1 + 0.25, y_{\text{last}}]$ where y_{last} is 0.25 + the location of the first 2-sensor in $[x_1, x_1 + 0.25]$ if there is any, or $y_{\text{last}} = x_1 + 0.5$ if there is no 2-sensor in $[x_1, x_1 + 0.25]$. The exact possible locations of the endpoint are discussed in the next subsection. Second, eliminate all the completely supplied sensors, and then repeat the process for the new distribution of sensors calling the leftmost uncovered sensor x_1. We will call the visited 2-sensors in $(x_1 + 0.25, y_{\text{last}}]$ *partially-supplied* sensors, since they are not completely supplied and will be addressed collaboratively with the next MC in an overlapping region.

What makes this problem hard is what we will call *the constraint of overlaps* which states that if two MCs supply a 2-sensor collaboratively in their overlapping region, then the coverage areas of the two MCs have to be equal. This natural constraint arises from the fact that if the two coverage areas are not equal, then the time gap between their visits to the 2-sensor in the overlapping area will keep changing until it eventually reaches more than 0.5, which would mean that the concerned 2-sensor in the overlapping area is not supplied properly.[1]

Determining the endpoint of the coverage area of any new deployed MC is the hardest part. We find that the possibilities of the best endpoint are limited: if there is no 2-sensor in $(x_1 + 0.25, y_{\text{last}}]$, then choosing the endpoint to be y_{last} will always be the best, but if there exists at least one 2-sensor in

[1] The time gap eventually becomes greater than 0.25 by reaching $min\{2 \times \text{Area}_1, 2 \times \text{Area}_2\}$ if $\frac{\text{Area}_1}{\text{Area}_2}$ is rational. If the ratio of their coverage areas is not rational, then it reaches $min\{2 \times \text{Area}_1, 2 \times \text{Area}_2\} - \delta,\ 0 < \delta < \epsilon\ \forall \epsilon > 0$.

Algorithm 1 Endpoint selection for MC

Input: Sensor locations $\{x_1, x_2, \dots, x_n\}$ and frequencies $\{f_1, f_2, \dots, f_n\},\ f_k \in \{1, 2\}$.
The length l of the previous MC's coverage area // $l = 0$ if there is none.
Output: The set of possible endpoints for the MC.
Depending on criterion $\mathcal{C}$, Assign $Possible\_Endpoints$ to be:
Case 1: $\{y_{\text{final}}, y_{\text{partition}}\}$.
Case 2: $\{x_1 + 0.25\}$.
Case 3: $\{x_1 + 0.25, x_1 + l\}$.

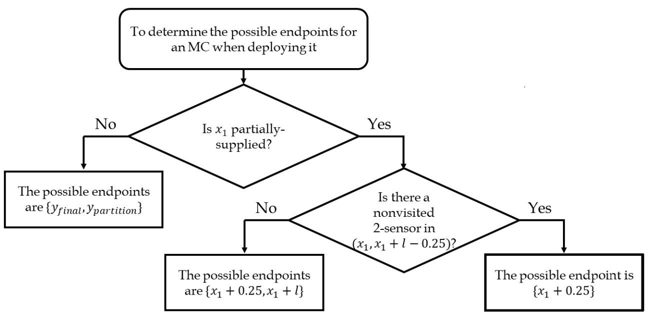

Fig. 4: An illustration of criterion $\mathcal{C}$ of how to determine the endpoints.

$(x_1 + 0.25, y_{\text{last}}]$ and x_1 is not a partially-supplied sensor, we will be left with two options:

- **Option 1**: Have all of the 2-sensors in $(x_1 + 0.25, y_{\text{last}}]$ supplied collaboratively with the next MC by having them in an overlap region between the two MCs.
- **Option 2**: **1)** Define $y_{\text{partition}}$ to be a point in $[y_{\text{first}}, y_{\text{last}}]$, where y_{first} is the first 2-sensor in $(x_1 + 0.25, y_{\text{last}}]$, **2)** have the 2-sensors in $[y_{\text{first}}, y_{\text{partition}}]$ supplied collaboratively with the next MC by having them in an overlap region between them, and **3)** have the 2-sensors in $(y_{\text{partition}}, y_{\text{last}}]$ supplied completely by the next MC.

Figure 1 shows the two options of the endpoints when deploying an MC in the case where the sensor x_1 is not partially-supplied.

Considering the *constraint of overlaps*, the always-best choice for the endpoint under option 1 will be y_{last} and the always-best choice under option 2 will be $y_{\text{partition}}$, where $y_{\text{partition}} = max(x_i) - l_{\text{covered}}$, x_i is a 2-sensor location in $[x_1 + 0.25, y_{\text{last}}]$ that satisfies the following condition: There is no 2-sensor in $(x_i - l_{\text{covered}}, x_i)$, and $l_{\text{covered}} = y_{\text{first}} - (x_1 + 0.25)$. However, if $max(x_i) = y_{\text{first}}$, we set $y_{\text{partition}} = y_{\text{first}}$, and if there is no 2-sensor in $(x_1 + 0.25, y_{\text{last}}]$, we set $y_{\text{partition}} = y_{\text{last}}$.

If the sensor x_1 is partially-supplied, then the endpoint of the next MC's coverage area will lie under two other options:

- **Option 3**: The endpoint is $x_1 + 0.25$ (we will not make use of the overlap.)
- **Option 4**: The endpoint is $x_1 + l$, where l is the coverage area of the previous MC (we will make use of the overlap.)

Figure 3, which illustrates the partially-supplied sensors in the red color, shows these two options. This means that when we deploy a new MC, there will be only one possible start point of its coverage area, which is the leftmost uncovered sensor, and a maximum of two possible options of its endpoint: option 1 and option 2 if there is no partially-supplied in the remaining distribution, or option 3 and option 4 if x_1 is partially-supplied.

Algorithm 2 Search space for the optimal solution $\mathcal{O}$

Input: Sensor locations $\{x_1, x_2, \dots, x_n\}$ and frequencies $\{f_1, f_2, \dots, f_n\},\ f_k \in \{1, 2\}$.
Output: The set of possible MC-solutions including $\mathcal{O}$.
Initialization: All sensors are unvisited.
$S = \{\}$ //The search space of the MC-solutions.
$l = 0$ //The last deployed MC's coverage area.
Optimal$(x_1, x_2, \dots, x_n, l, S)$:
if all sensors are completely supplied **then**
$S = S \cup \{$Last generated MC-solution$\}$.
return
Call Algorithm 1 to determine $Possible\_Endpoints$.
for each k in $Possible\_Endpoints$ **do**
Generate an MC that covers $[x_1, k]$, add it to the current MC-solution, and let $l = k - x_1$.
Eliminate all sensors in $[x_1, x_1 + 0.25]$ and 1-sensors in $[x_1 + 0.25, k]$.
Annotate the 2-sensors in $(x_1 + 0.25, k]$ as 'visited'.
Call **Optimal**$(x_1, x_2, \dots, x_n, l, S)$ where x_1 is the leftmost sensor.

Furthermore, in the case of having partially-supplied 2-sensors in the remaining distribution, choosing option 4 in the special case where there is a nonvisited 2-sensor in $(x_1, x_1 + l - 0.25)$ will result in a solution that does not have property 3. Hence, we exclude option 4 in this case.

Now, we have everything set up to define a criterion $\mathcal{C}$ to choose the set of possible endpoints for the MC from three cases: if x_1 is not partially-supplied, there will be two possible endpoints (option 1 and option 2), if x_1 is partially-supplied and there is a nonvisited 2-sensor in $(x_1, x_1 + l - 0.25)$, then there will be one possible endpoint (option 3), while there will be two possible endpoints (option 3 and option 4) if there is no 2-sensor in $(x_1, x_1 + l - 0.25)$. Figure 4 shows criterion $\mathcal{C}$.

Algorithm 1 produces the set of the possible endpoints for any new MC to be deployed to cover an area starting from the leftmost remaining uncovered sensor x_1.

C. Algorithm design

At this point, we tackled the hardness of the problem: *Where should the endpoint of the coverage area of the next MC be determined?* Even though we know for sure where the MC's coverage area starts (it will start from the leftmost uncovered sensor), determining where the previous one ends remains hard. Algorithm 1 is designed so that the produced MC-solutions hold the properties of our reduced search space.

The main question that holds and forces us to exhaust all possible MC-solutions is *how can we make our next deployed*

Algorithm 3 Greedy 1.5-approximation solution

Input: Sensor locations $\{x_1, x_2, \dots, x_n\}$ and frequencies $\{f_1, f_2, \dots, f_n\}$, $f_k \in \{1, 2\}$.
Output: A 1.5-approximation MC-solution.
Initialization: $i = 0$ //The MCs' indexes.
1: **While** there is a non-zero leftmost sensor x **do**
2: $i = i + 1$.
3: **if** there is a leftmost 2-sensor x' in $[x, x + 0.25]$ **then**
Generate MC_i that covers $[x, x' + 0.25]$.
4: **else**
Generate MC_i that covers $[x, x + 0.5]$.
5: Eliminate the sensors in $[x, x + 0.25]$.
6: Subtract 1 from visited sensors in $(x + 0.25, x + 0.5]$.
7: for every MC_{2i}, generate an additional MC that covers the same area MC_{2i} covers.

MC contribute in supplying the other sensors in the $(1, 2)$-WSN in a way that makes the remaining distribution of sensors need as less MCs as possible? Algorithm 2 exhausts all the possible MC-solutions with properties 1–3.

Theorem 1. *The MC-solution in S, which is produced by Algorithm 2, with the least number of MCs is the optimal solution $\mathcal{O}$ and has a time complexity of $O(d \times 16^L)$.*

Proof. Algorithm 2 does nothing but exhaust all of the MC-solutions with properties 1–3 in our subspace. Since $\mathcal{O}$ has these properties, choosing the MC-solution with the least number of MCs from all of the generated solutions with these properties gives us $\mathcal{O}$. □

We conjecture that the $(1, 2)$-WSN problem is an NP-hard problem. Analysing the complexity of our brute-force algorithm (Algorithm 2) shows that the recursive call inside the loop is equivalent to a maximum number of nested loops of $L/0.25$, where every loop has a maximum of two iterations. Each iteration takes $O(n/0.25)$ to find $max(x_i)$ in order to calculate $y_{\text{partition}}$, where n is the number of sensors in the search region of $max(x_i)$. This search region is bounded by 0.25. This means that in the worst-case scenario, the algorithm takes $O(d \times 2^{L/0.25}) = O(d \times 16^L)$ time, where d is the maximum sensor-density of any region of length 0.25 in the given linear WSN. d is bounded by $n/0.25$.

IV. Approximation Solutions for $(1, 2)$-WSN

In this section, we propose our new greedy approximation algorithm and an enhancement for it in the first part, and perform an analytical expansion for a previous general approximation algorithm in the second part.

A. A novel approximation algorithm

First, we will set up a lower bound for the optimal solution $\mathcal{O}$. The lower bound is going to be determined by seeking the optimal solution Ω of the problem after lifting *the constraint of overlaps*. We will assume that 2-sensors are now satisfied if they are supplied collaboratively by two MCs of different

Algorithm 4 General greedy solution

Input: Uncovered sensor locations $\{x_1, x_2, \dots, x_n\}$ and frequencies $\{f_1, f_2, \dots, f_n\}$, $f_k \in \mathbb{R}$.
Output: A 2-approximation MC-solution.
if $n = 0$ **then return**.
Generate an MC that goes back and forth as far as possible at a full speed to cover sensors at $\{x_1, \dots x_{i-1}\}$.
Recursively call Algorithm 4 for $\{x_i, \dots x_n\}$.

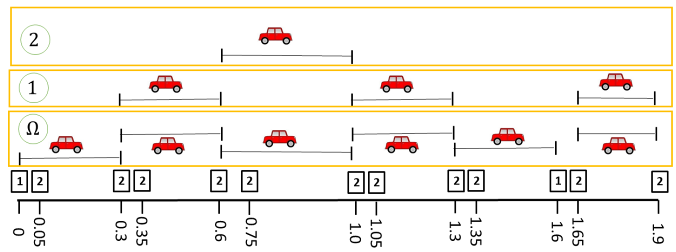

Fig. 5: The lower bound of the optimal, the 1.5-approximation algorithm, and the enhanced 1.5-approximation algorithm.

coverage areas, these coverage areas shall not exceed 0.5. Then we will propose an approximation solution for the original problem which produces a number of MCs that is bounded by 1.5 of the lower bound produced by Ω.

Even after lifting *the constraint of overlaps*, the optimal solution of the resulting alleviated problem Ω still has the properties 1–3. The optimal solution Ω is produced as follows: deploying an MC covering the area starting from the leftmost sensor and ending as far as possible while completely supplying the sensors in the first 0.25 distance. We may treat the 2-sensors visited by this sensor but not completely supplied (the visited 2-sensors after the 0.25 distance) as 1-sensors for the next MC. Continuing to deploy the MCs at this manner produces the lower bound of the optimal solution Ω.

Theorem 2. *The MC-solution, which is produced by Algorithm 3, is an approximation solution with ratio of 1.5 and has a time complexity of $O(L)$.*

Proof. After producing Ω, simply addressing the sensors in the overlapping regions (i.e., considering the *the constraint of overlaps* again) by deploying additional MCs for them gives us the 1.5-approximation solution. Algorithm 3 produces this solution; lines 1–6 of it produce Ω, line 7 generates the additional MCs that address any possible overlap. The number of these additional MCs cannot exceed half the number of MCs in Ω. This confirms our approximation ratio of 1.5.

Analysing the time complexity of Algorithm 3 shows that its run-time is linearly proportional to $L/0.25$ as the number of iterations is upper-bounded by $O(L/0.25) = O(L)$, where L is the whole length of the linear WSN. □

We may enhance the solution produced by Algorithm 3 by deploying the additional MCs only when needed instead of deploying them for all even-numbered MCs. That may be done by making the last line of the algorithm produce additional

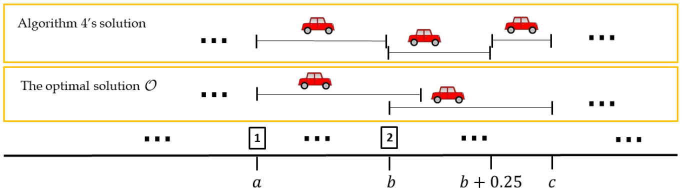

Fig. 6: The optimal solution $\mathcal{O}$ and Algorithm 4's approximation solution.

MCs to only cover the overlaps of different-lengths coverage areas. Even though this improvement generally reduces the number of additional MCs, the approximation factor of the solution remains 1.5.

Figure 5 shows the MCs generated by lines 1–6 of the approximation algorithm (Ⓘ), the additional MCs added by line 7 (①), and the additional MCs added by the enhanced approximation algorithm which only assigns MCs to address the overlaps between two different length coverage areas (②).

B. New analysis for the 2-approximation solution

Wu *et al.* [1] have proposed Algorithm 4 as an approximation algorithm to generate an MC-solution for heterogeneous WSNs with any frequencies. They have proved that this algorithm produces a solution with an approximation ratio of 2. In this section here, we prove that this approximation ratio becomes tighter as it reaches 1.5 for $(1, 2)$-WSNs.

Theorem 3. *The MC-solution, which is produced by Algorithm 4, is an approximation solution with ratio of* 1.5 *when applied to the* $(1, 2)$*-WSN and has a time complexity of* $O(L)$.

Proof. Considering the optimal solution $\mathcal{O}$, we know from property 2 that the optimal solution $\mathcal{O}$ may have overlaps between the coverage areas of no more than two MCs. It is trivial to show that if, for some distribution of sensors, $\mathcal{O}$ has no overlaps, then Algorithm 4 (as well as Algorithm 3) produces the same optimal solution. However, when the optimal solution $\mathcal{O}$ gets to have two MCs with an overlapping region as shown in Figure 6, Algorithm 4 produces exactly three MCs in order to be able to cover the same region the two overlapping MCs cover.

The three MCs will be produced by Algorithm 4 in the following order: The first MC will be deployed to cover the sensors in the region $[a, b)$, then the second one will be deployed to cover the sensors in the region $[b, b + 0.25]$. The third MC will cover any remaining sensor in the region $(b + 0.25, c]$.

In the worst-case scenario, when the optimal solution O has each MC to cover a region with an overlap with exactly one other MC, where this region is significantly far from other coverage areas, Algorithm 4 produces three MCs for each two overlapping MCs. Hence, an approximation ratio of 1.5.

The run-time of the algorithm is upper-bounded by $L/0.25$; for any WSN with sensors of frequencies of 1's and 2's and a given length L, the number of iterations will not exceed $\lceil L/0.25 \rceil$, even for a dense distribution of 2-sensors. This upper bound of the worst-case scenario gives us a fairly tight upper bound for the time complexity of Algorithm 4, which is $O(L)$. □

V. SIMULATION

In this section, we conduct simulations to evaluate the effectiveness of the algorithms discussed in this paper.

A. Experimental Settings

In our simulations, the frequencies of the sensors (f_k) follow the Bernoulli distribution with a certain probability for each of the two possible frequencies to occur. The distribution of the locations of the sensors was considered to follow one of three different scenarios. The first scenario distributes the sensors uniformly on the given line segment of length l after placing a sensor on each edge of the line segment. We will call this distribution the *uniform distribution* of sensors.

The second distribution of the locations is cluster distribution, where k sensors are distributed uniformly as the centers of the clusters, then two sensors are placed on the two edges of the line WSN of length l, then the remaining sensors are divided equally into k groups (if there is a shortage in the remaining number of sensors to be divided equally into k groups, all the shortage is applied to one random cluster). The remaining sensors are distributed around the k center of clusters on a normal distribution $N(x_{\text{cluster}}, \sigma^2)$, where each group of sensors has its own x_{cluster} as the mean of their locations, and a certain standard deviation σ. We will call this distribution the *cluster distribution* of sensors.

The third distribution is a mixture between the latter two distributions, where a certain percentage of sensors are distributed to follow the uniform distribution, while the rest of them are distributed to follow the cluster distribution. We will call this distribution the *mixed distribution* of sensors.

Our choice to choose such distributions comes from their practical emulation of real-world situations, whether for WSNs or border patrolling applications. This gives us a vast number of parameters: the first one is the percentage of the 2-sensors, the second one is the probability distribution which the locations of the sensors follow. Each one of those distributions has its own additional parameters. The uniform distribution has an additional two parameters: the length of the WSN and the total number of sensors. The cluster distribution has an additional four parameters: the length of the WSN, the total number of sensors, the number of clusters, and the standard deviation of the sensors in the clusters. The mixed distribution has an additional five parameters: the length of the WSN, the total number of sensors, the number of clusters, the standard deviation of the sensors in the clusters, and the percentage of the uniformly distributed sensors.

B. Algorithm Comparison

We consider various settings to compare the performance of the four algorithms: the algorithm that produces the optimal solution $\mathcal{O}$, our proposed 1.5-approximation greedy algorithm (Algorithm 3), the enhanced version of our 1.5-approximation

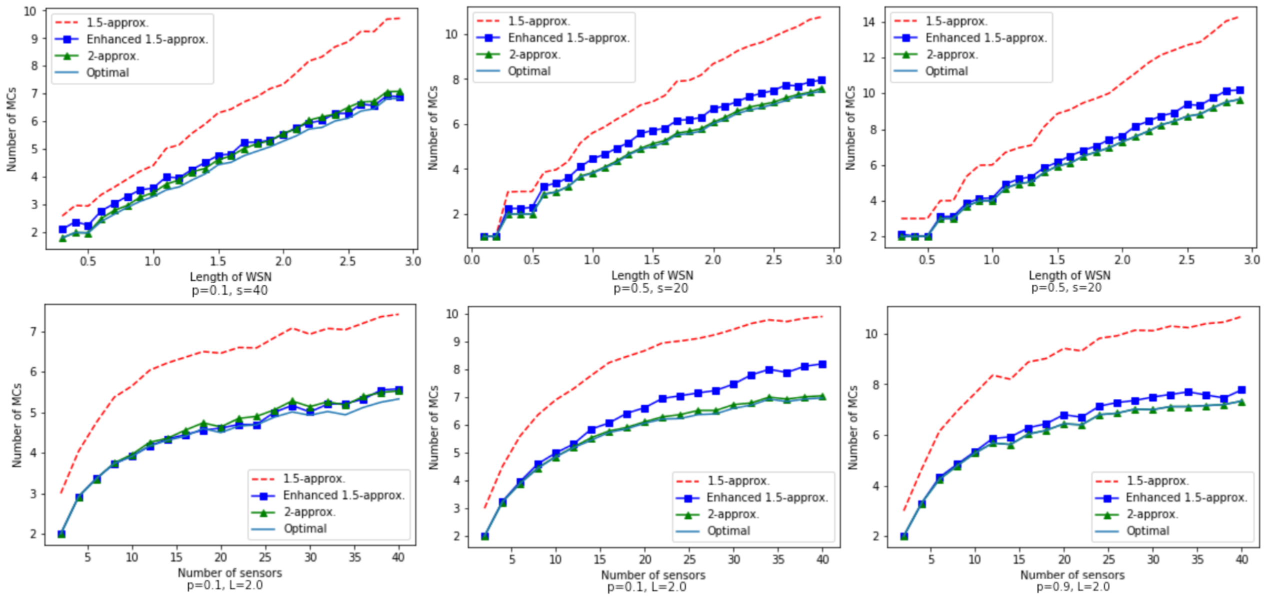

Fig. 7: The results of the algorithms under the uniform distribution with various percentages of 2-sensors. s is the number of sensors, p is the percentage of the 2-sensors, and L is the length of the WSN.

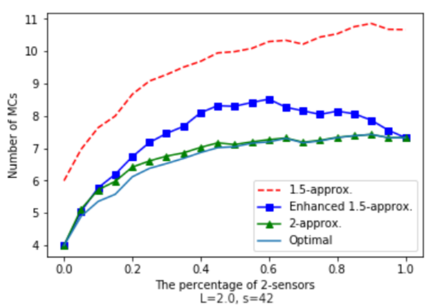

Fig. 8: The behavior of the algorithms with uniform distribution under varying percentage of 2-sensors and fixed other parameters.

algorithm, and Wu's greedy algorithm for general line heterogeneous WSNs (Algorithm 4). Because of the exponential time-complexity of our optimal algorithm, we include small-scale scenarios of limited lengths and numbers of sensors.

C. Experimental Results

We can observe from the first three plots in Figure 7 that for the uniform distribution, fixing all of the parameters but the length of the WSN shows that the number of MCs grows almost linearly with the increase of the length of the WSN. The enhanced algorithm and the Wu's 2-approximation general algorithm both give results very close to the optimal solution under a constant number of sensors. Furthermore, it is clear that the dominance by 1-sensors favors the enhanced 1.5-approximation algorithm over the 2-approximation algorithm.

Observing the last three plots in Figure 7 in which the density of the sensors varies under a fixed length of the WSN, we see that the 2-approximation algorithm behaves very closely to the optimal algorithm with different percentages of 2-sensors as opposed to the enhanced algorithm that depends significantly on the percentage of the 2-sensors deployed.

Figure 8 shows how the different algorithms behave for a fixed number of sensors and a fixed length of the WSN. At the two edges where we have homogeneous WSN, the enhanced and 2-approximation algorithms give exactly the same result as the optimal solution. In general, the 2-approximation algorithm performs better under these settings and very close to the optimal algorithm. However, for the settings where we have 1-sensors dominate the 2-sensors, the enhanced algorithm very closely beats the 2-approximation algorithm. The normal 1.5-approximation algorithm performs very closely to its upper bound due to the blind deployment of the additional MCs to address the overlaps whether they are needed or not.

We may observe from Figure 9 that the behavior of the algorithms under the cluster distribution, the first three plots show that, under low standard deviation (dense clusters), the enhanced and the 2-approximation general algorithms perform very closely to the optimal algorithm. As the clusters become more loose, the enhanced algorithm performs more poorly than the 2-approximation algorithm which does not get affected heavily by the standard deviation parameter. The third plot shows behavior close to the behavior in uniform distribution due to the high standard deviation.

The second three plots in Figure 9 show how the number of clusters affects the needed numer of MCs under different standard deviations; the tighter the clusters are (have lower standard deviation), the more their number correlates more strongly with the needed number of MCs. For low standard deviation, the number of MCs increases almost-linearly with the number of clusters under fixed other parameters. With a higher standard deviation, an increase in the number of

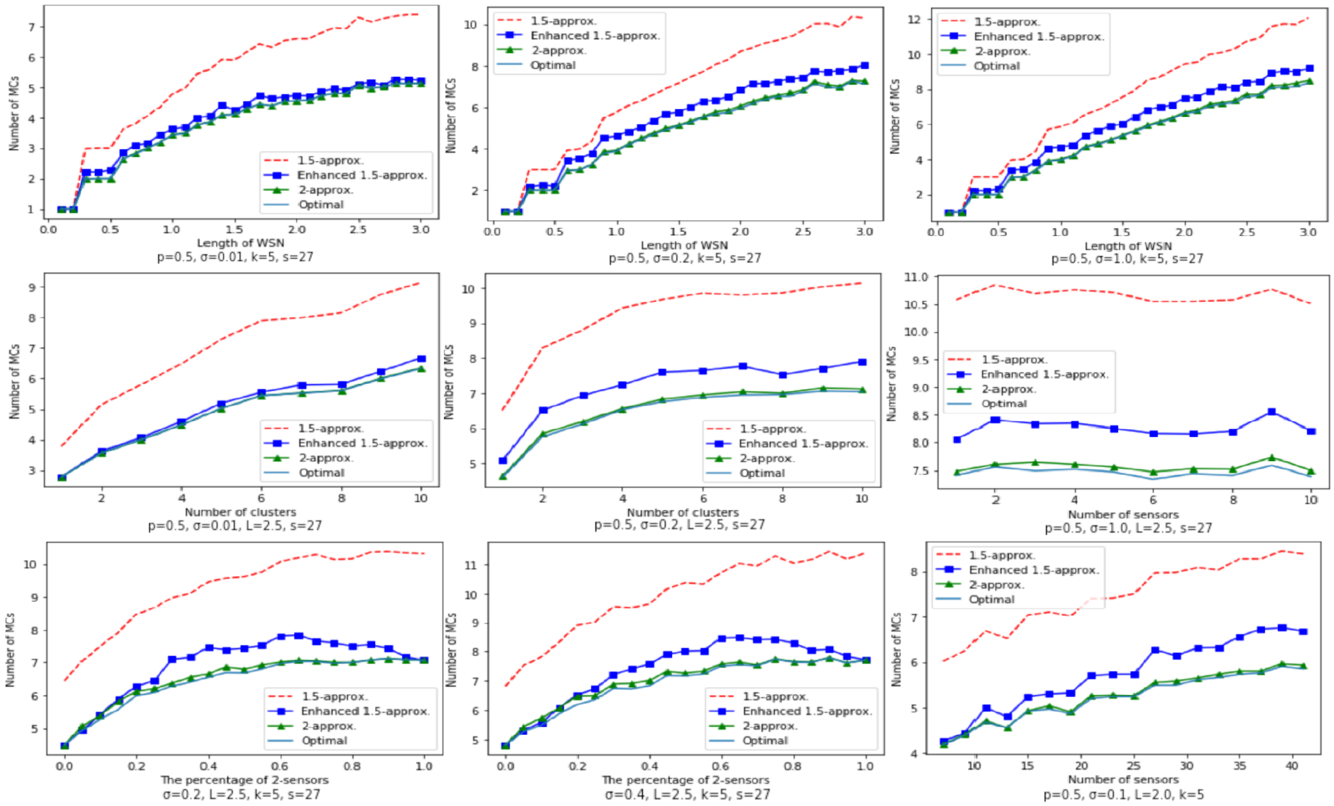

Fig. 9: The results of the algorithms under the cluster distribution with various standard deviations. p is the percentage of the 2-sensors, σ is the standard deviation, L is the length of the WSN, k is the number of clusters, and s is the number of sensors.

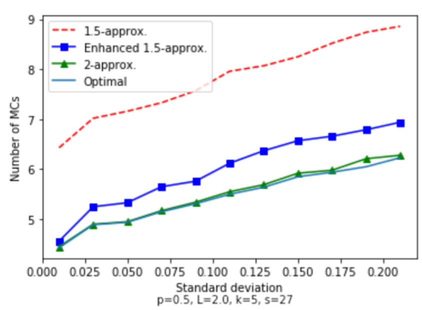

Fig. 10: The behavior of the algorithms with cluster distribution under varying standard deviation with fixed other parameters.

clusters affects the needed number of MCs less. For very high standard deviation values, the number of clusters does not affect the outcome of the algorithms. Furthermore, it is clear that the 2-approximation algorithm outperforms the enhanced 1.5-approximation algorithm under the cluster distribution. The last line of plots in Figure 9 shows again how the 2-approximation algorithm performs very closely to the optimal solution. This is due to the fact that this algorithm is actually exactly the same as the first six lines of Algorithm 3 (which produce the lower bound of the optimal solution Ω) except that in line 6, it does not subtract 1 from the 2-sensors in $(x + 0.25, x + 0.5]$.

Figure 10 shows how the number of MCs, for a fixed number of clusters with fixed length of WSN and number of sensors, correlates directly with the standard deviation, and how under various standard deviations, the 2-approximation algorithm remains very close to the optimal solution.

In Figure 11, the behavior of the algorithms under the mixed distribution is shown. The first plot shows that the length of the WSN affects the number of MCs almost as linearly as the previous two distributions. From the different distributions and various parameters settings, the previously proposed general 2-approximation algorithm outperforms the enhanced 1.5-approximation algorithm by around 10%, where the original 1.5-approximation algorithm produces a solution that is always close to the upper bound of 1.5. The 2-approximation algorithm remains very close to the optimal solution in all the observed scenarios due to its closeness to the algorithm that produces the optimal solution's lower bound.

VI. Conclusion

In this paper, we establish an investigation for the NP-hardness boundaries between the tractable and intractable solutions of the mobile charger coverage problem. The schedule of the least possible number of mobile chargers for heterogeneous line wireless sensor networks with sensors of frequencies 1's and 2's is studied. We obtain the optimal solution for this problem by exhausting all of the possible

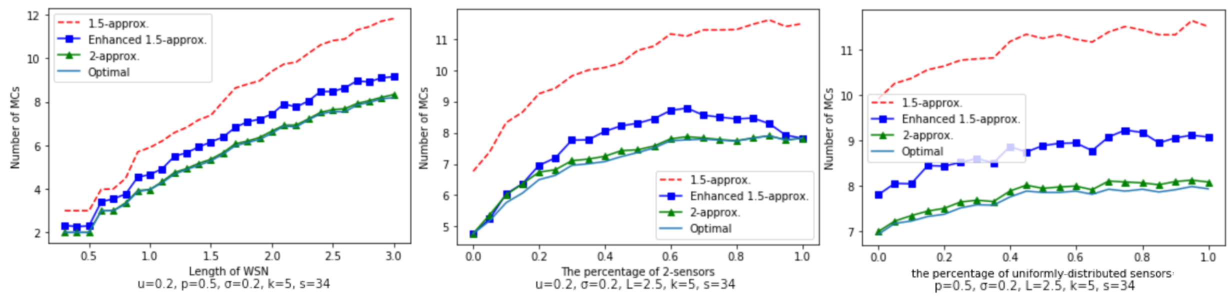

Fig. 11: The results of the algorithms under the mixed distribution with different settings, u is the percentage of uniformly-distributed sensors, p is the percentage of the 2-sensors, σ is the standard deviation, L is the length of the WSN, k is the number of clusters, and s is the number of sensors.

solutions with specific properties, and conjecture the NP-hardness of it. A 1.5-approximation algorithm, an enhancement of this approximation, and an analytical expansion for a previously proposed general 2-approximation algorithm are done. Simulation results compare the optimal solution with our approximation algorithms and the previous general approximation algorithm. The simulation shows that in practical set-ups, our enhanced algorithm performs 10% less than the 2-approximation algorithm, which remains very close to the optimal solution. In future work, we will try to study different line WSNs of different frequency ranges, prove the NP-hardness of this problem, and come up with better approximations.

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